# The first 10 years of Curve25519

Daniel J. Bernstein

University of Illinois at Chicago &
Technische Universiteit Eindhoven

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2005.05.19: Seminar talk; design+software close to done.

2005.09.15: Software online.

2005.09.20: Invited talk at ECC.

2005.11.15: Paper online; submitted to PKC 2006.



Abstract: "This paper explains the design and implementation of a high-security elliptic-curve-Diffie-Hellman function achieving record-setting speeds: e.g., 832457 Pentium III cycles (with several side benefits: free key compression, free key validation, and state-of-the-art timing-attack protection), more than twice as fast as other authors' results at the same conjectured security level (with or without the side benefits)."

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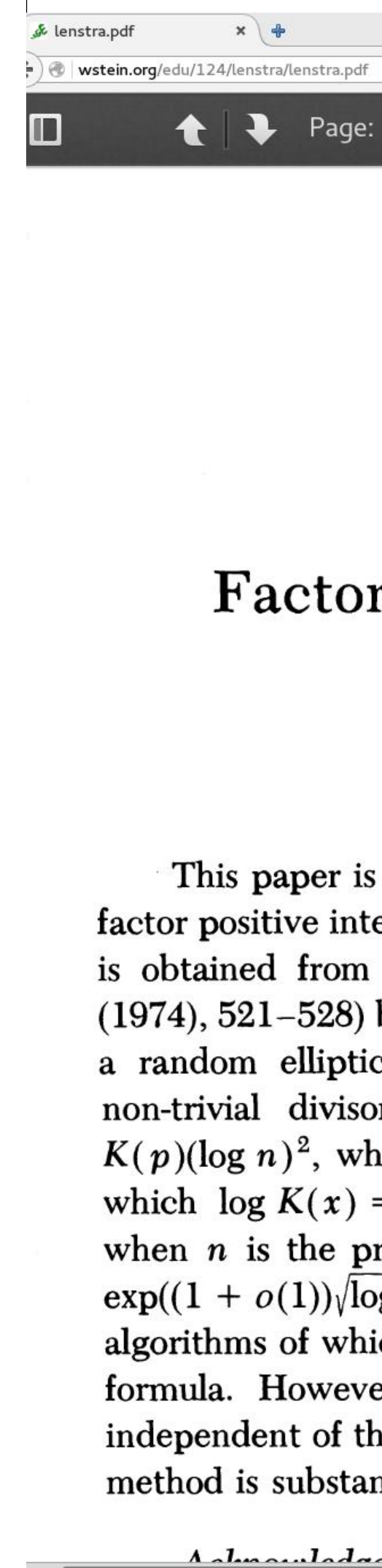
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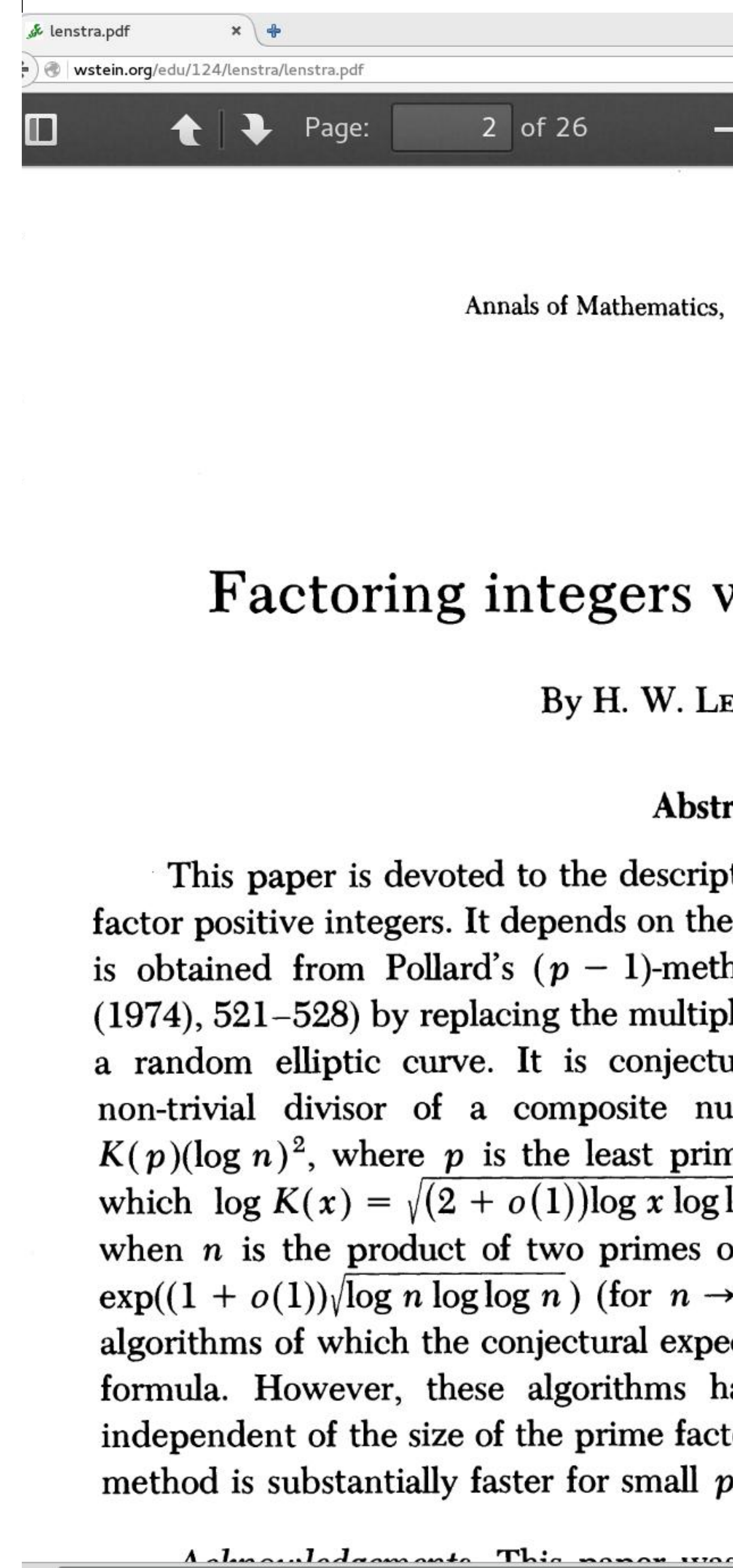

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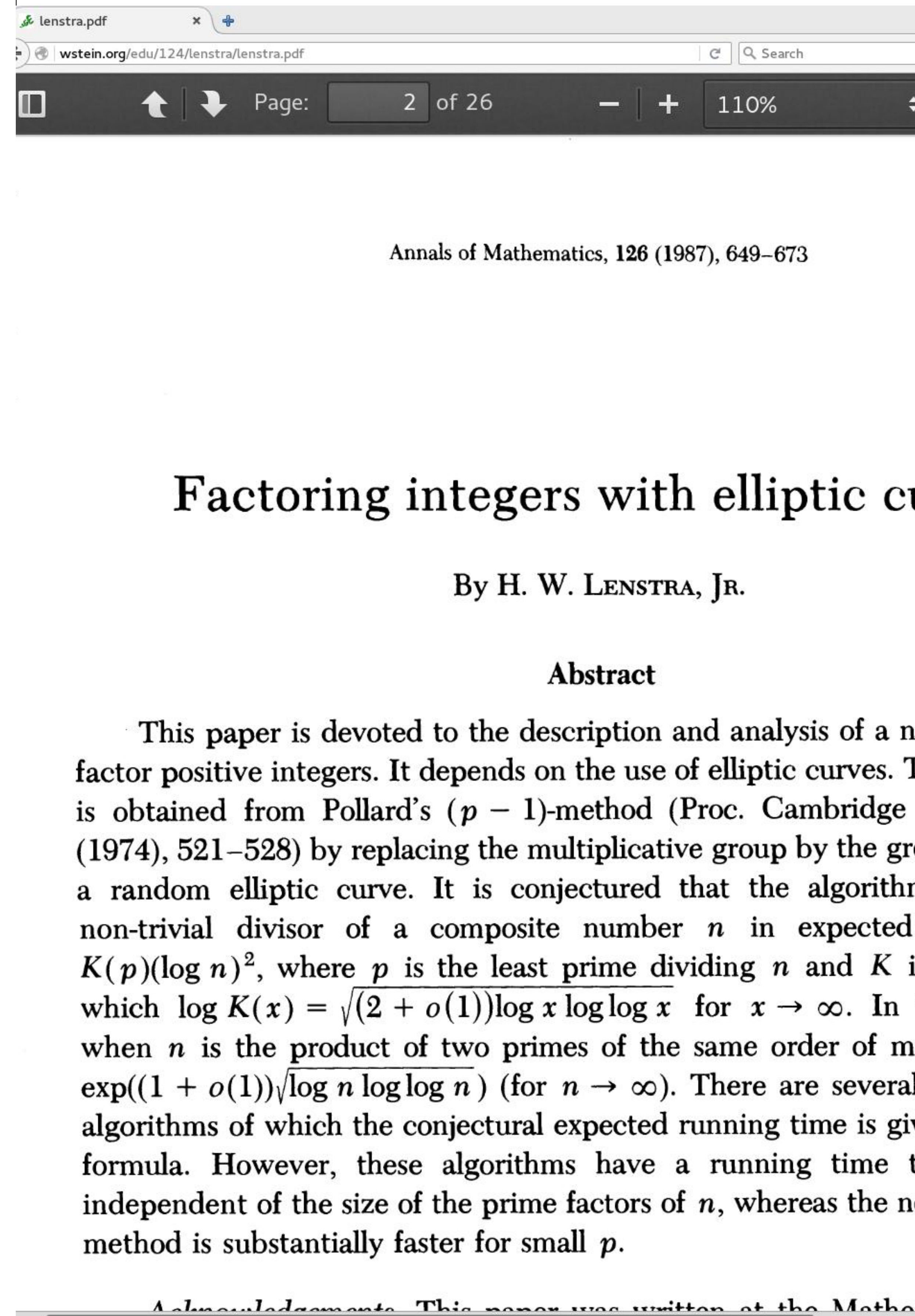

Annals of Mathematics, 126 (1987), 649–673

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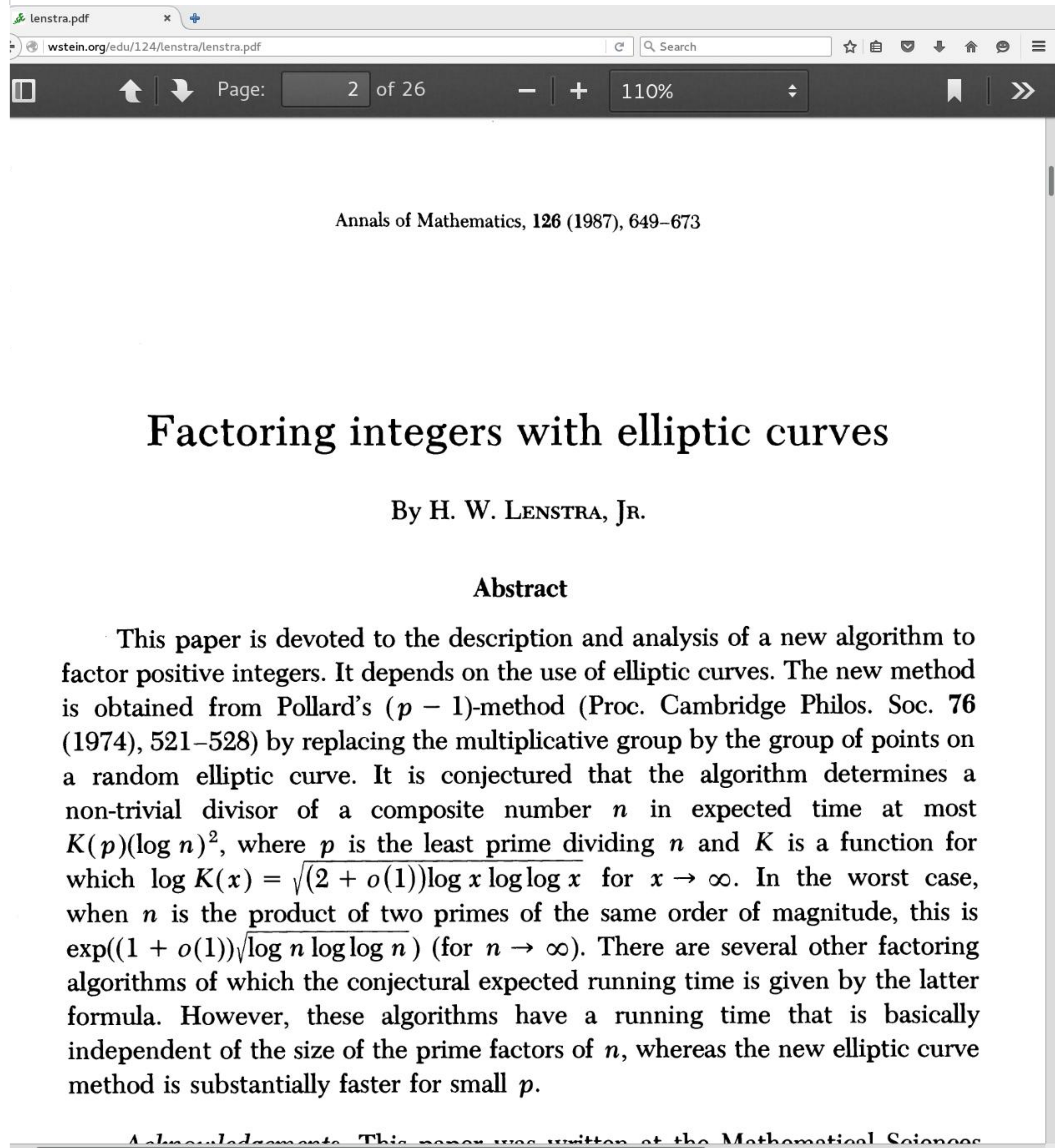


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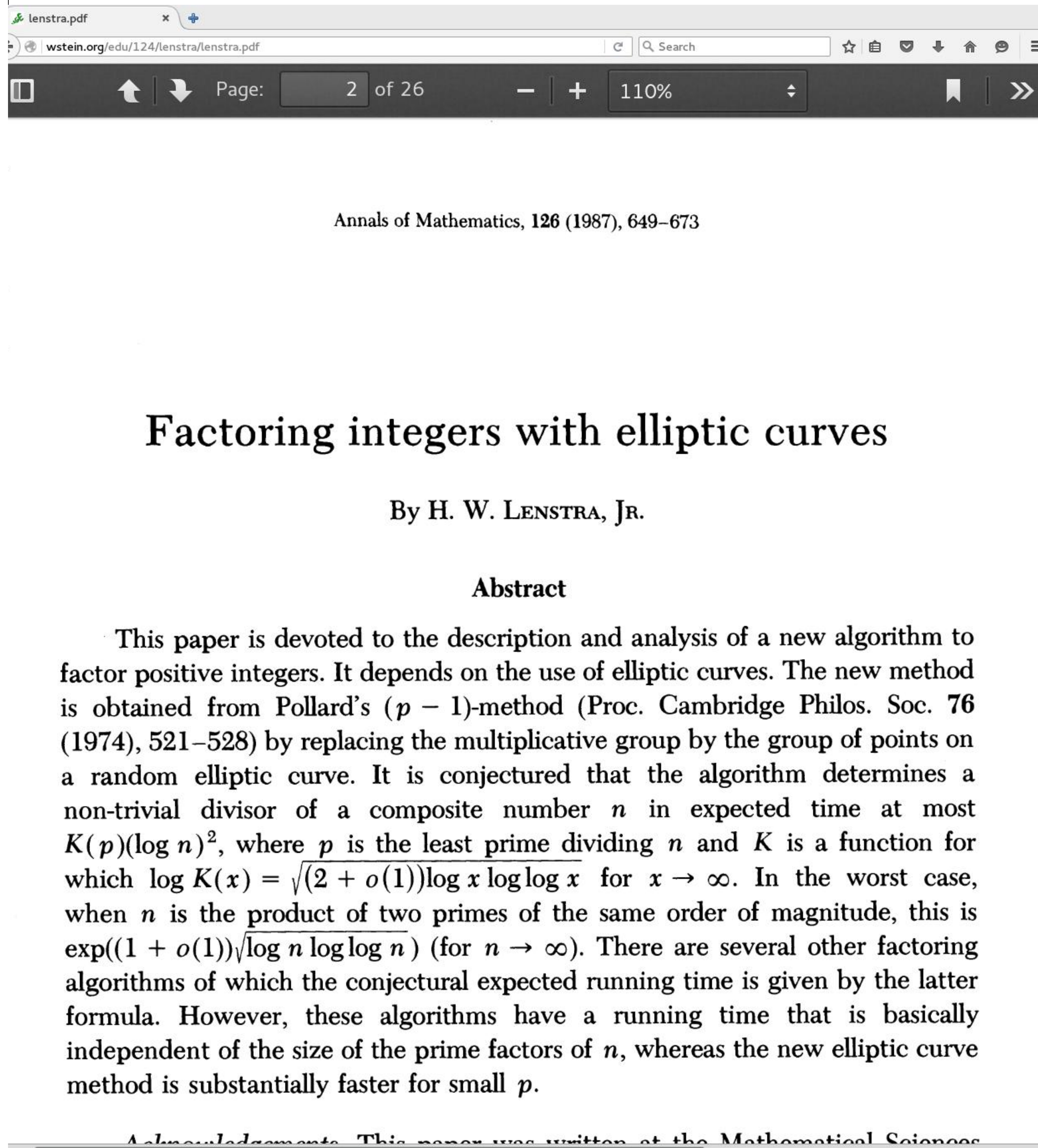


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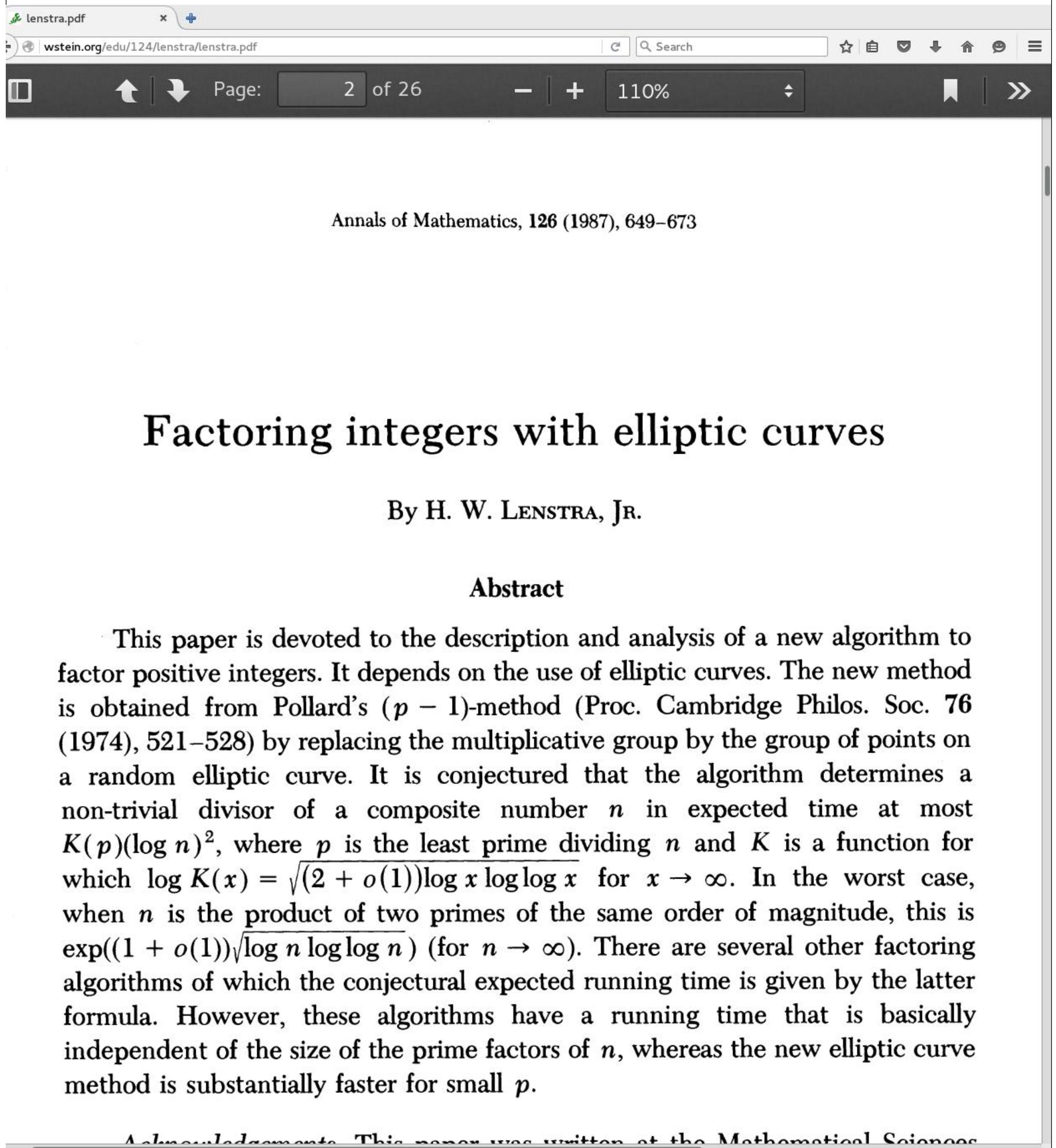


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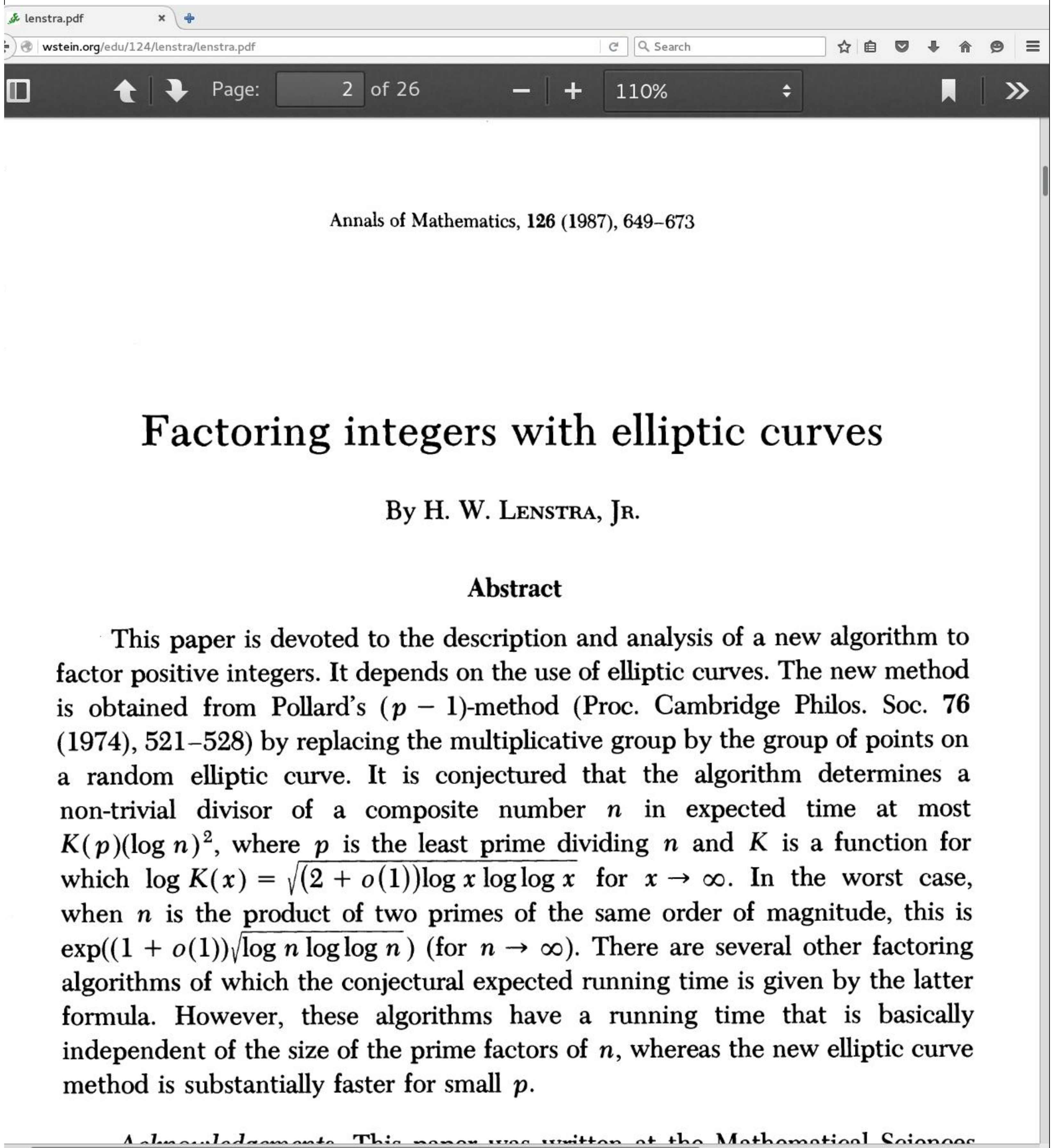


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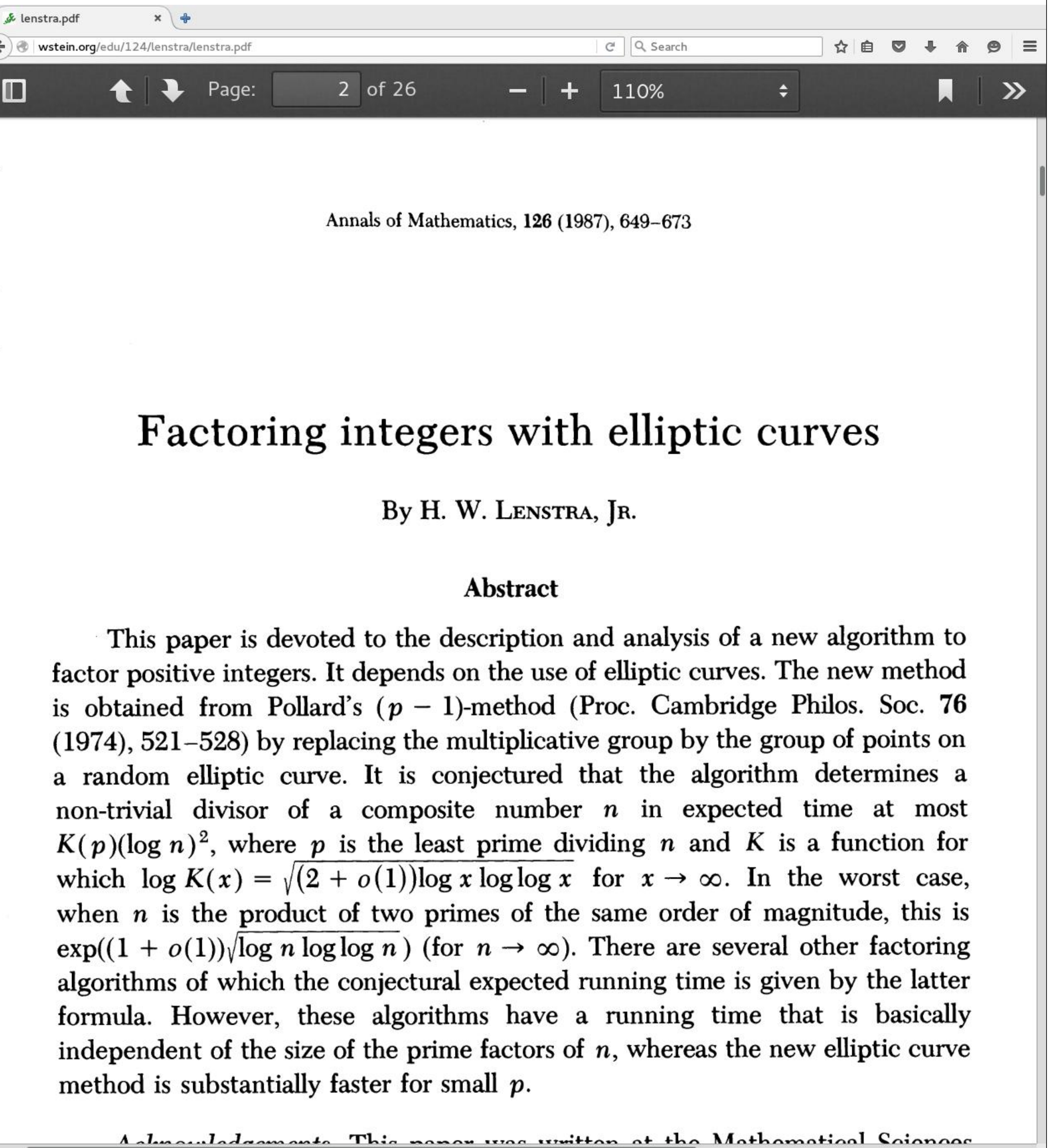
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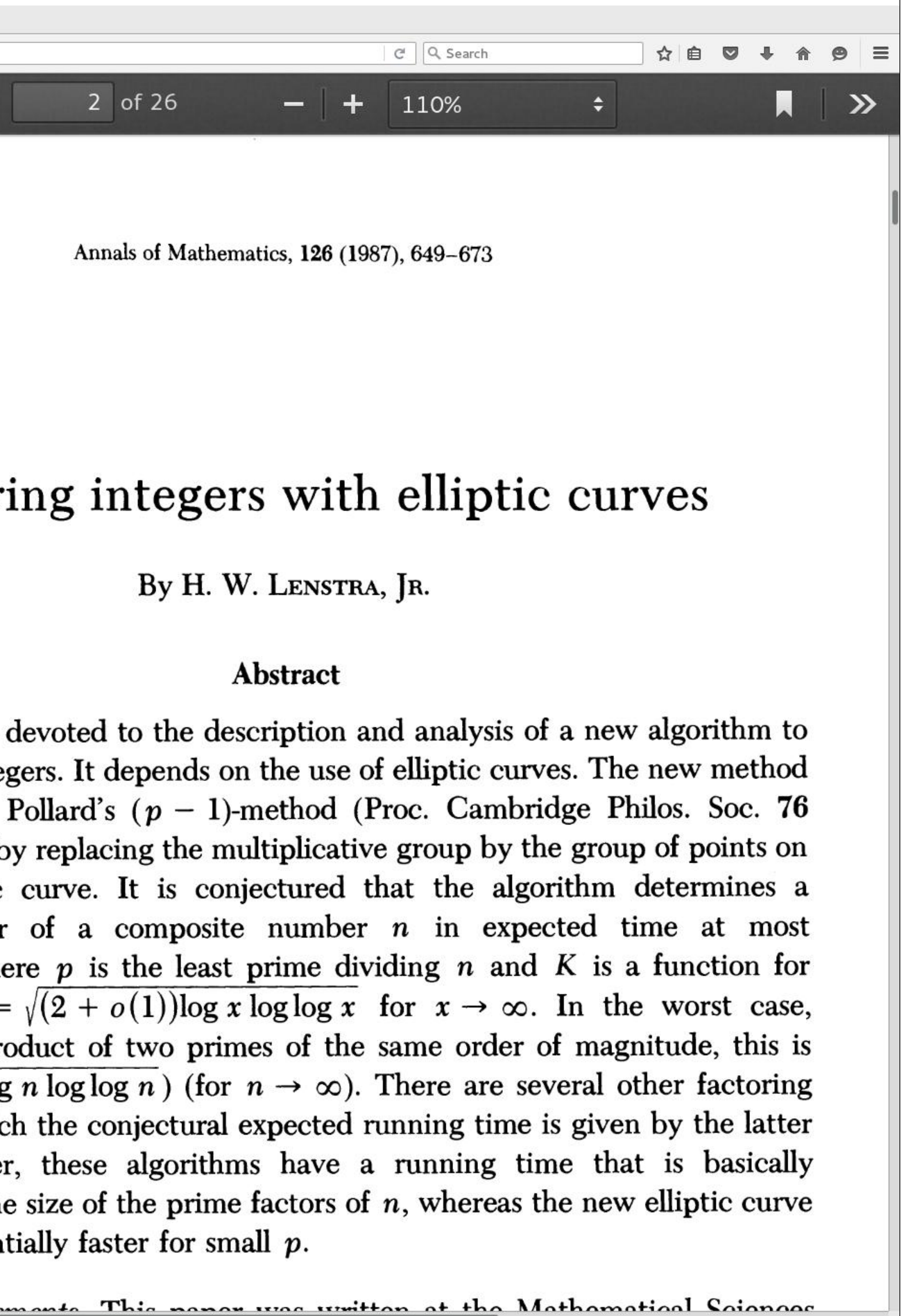
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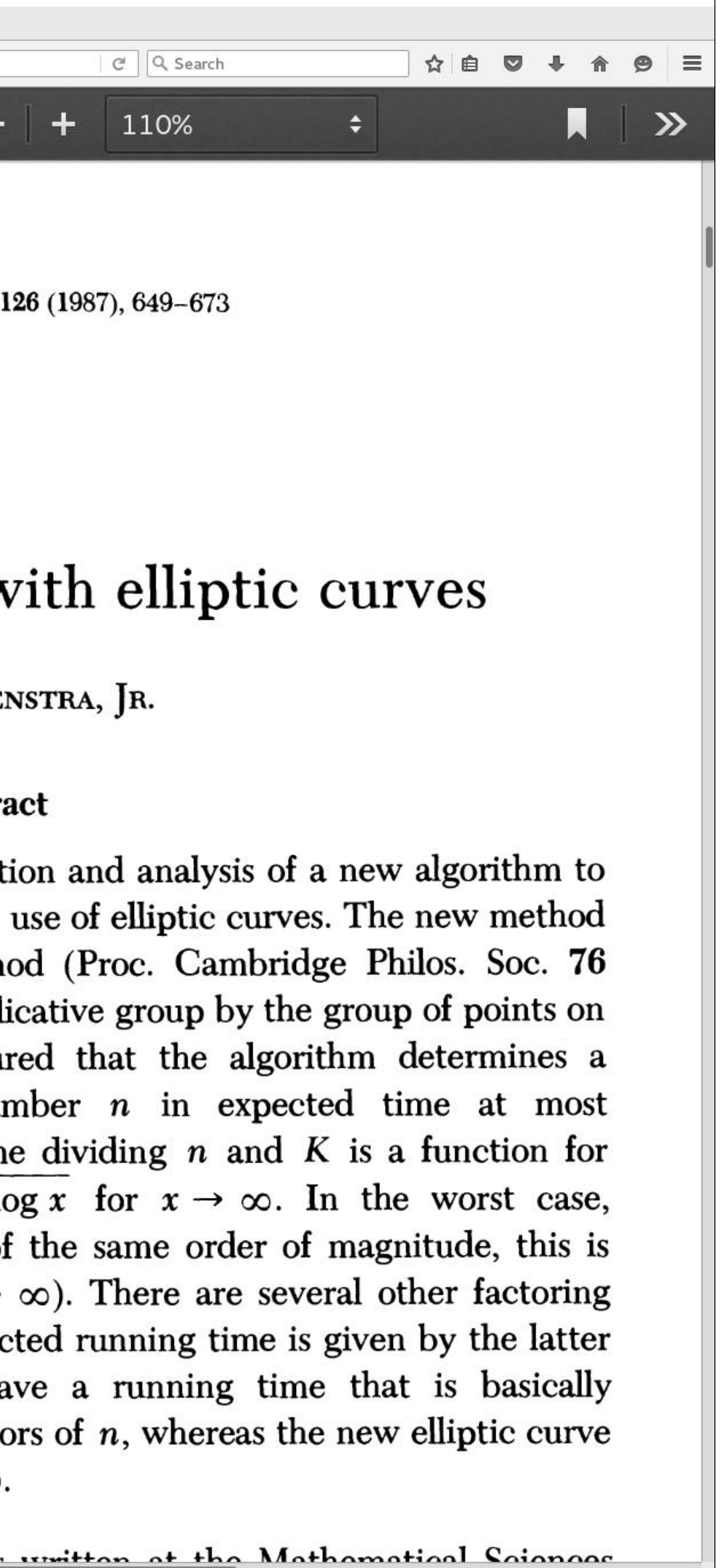


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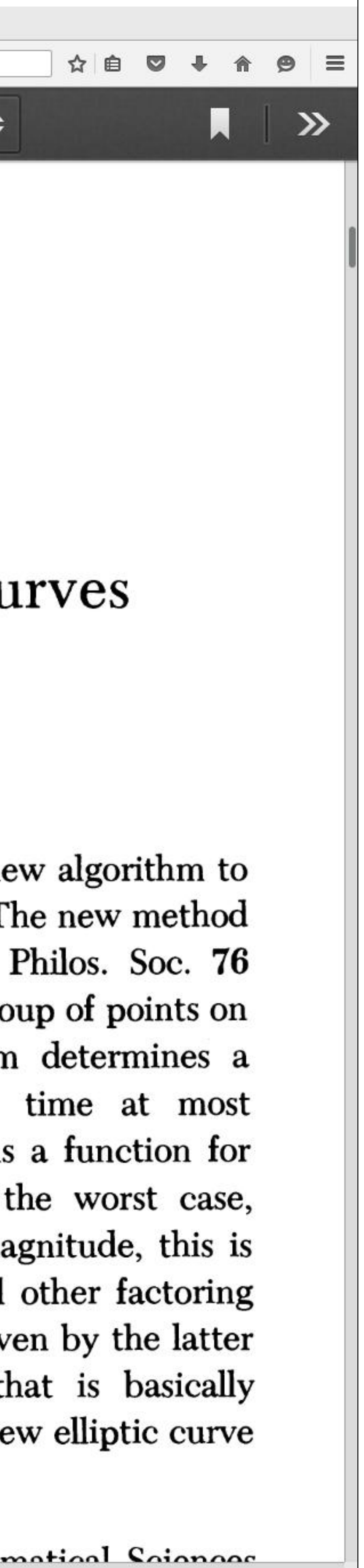
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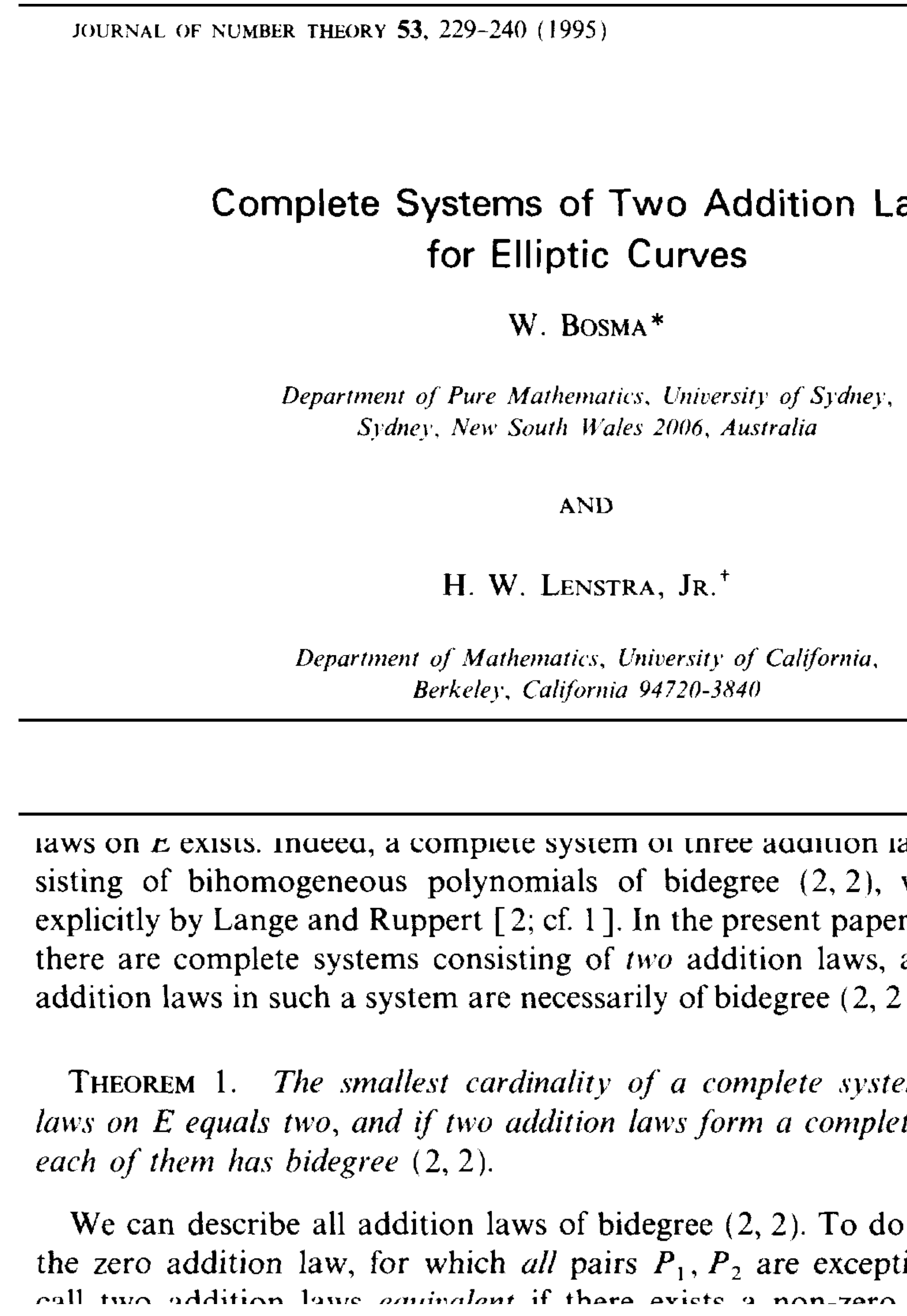
JOURNAL OF NUMBER THEORY 53, 229–240 (1995)

Complete Systems of Two Addition La
for Elliptic Curves

W. BOSMA*

*Department of Pure Mathematics, University of Sydney,*
*Sydney, New South Wales 2006, Australia*

AND

H. W. LENSTRA, JR.†

*Department of Mathematics, University of California,*
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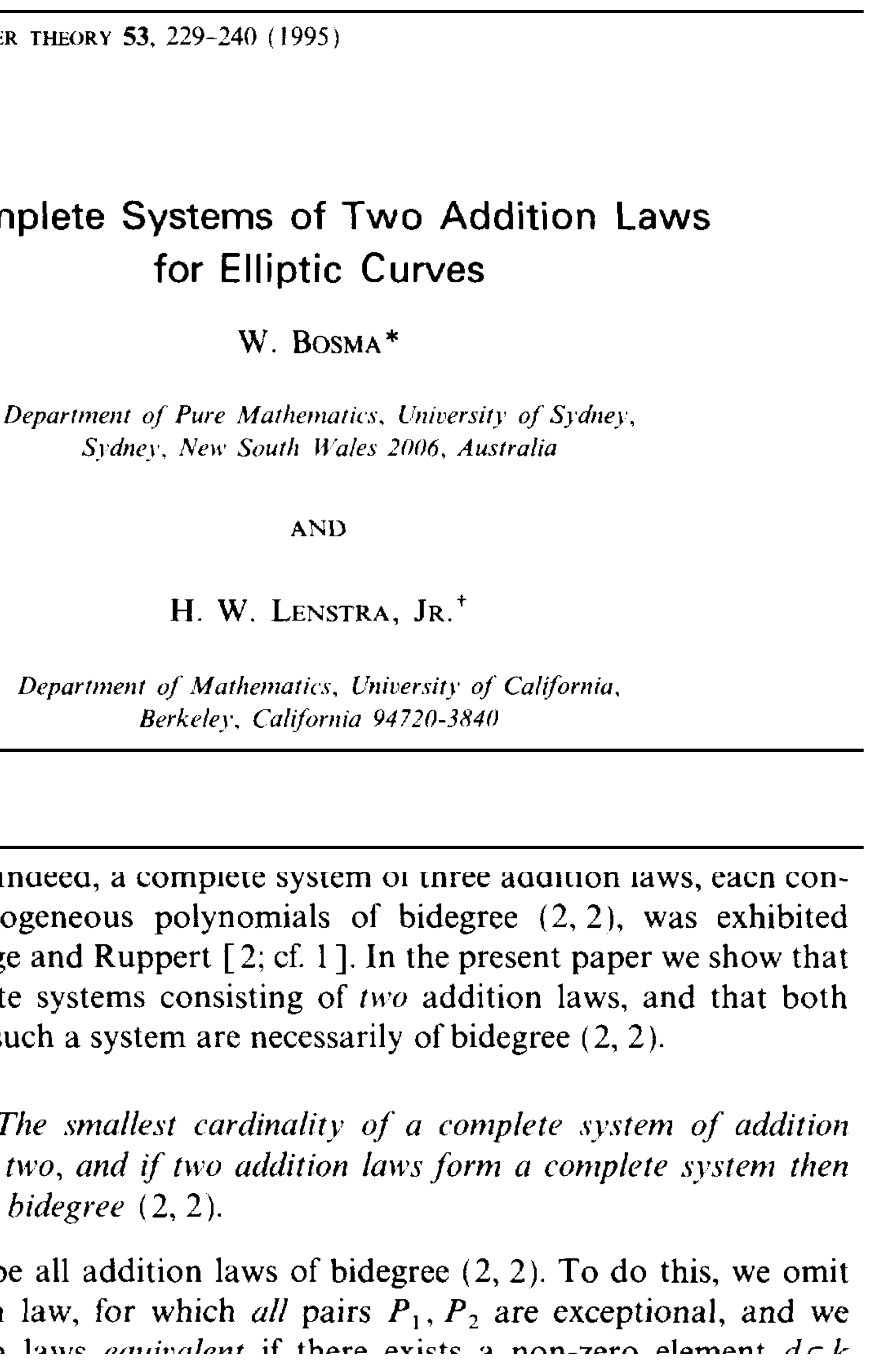
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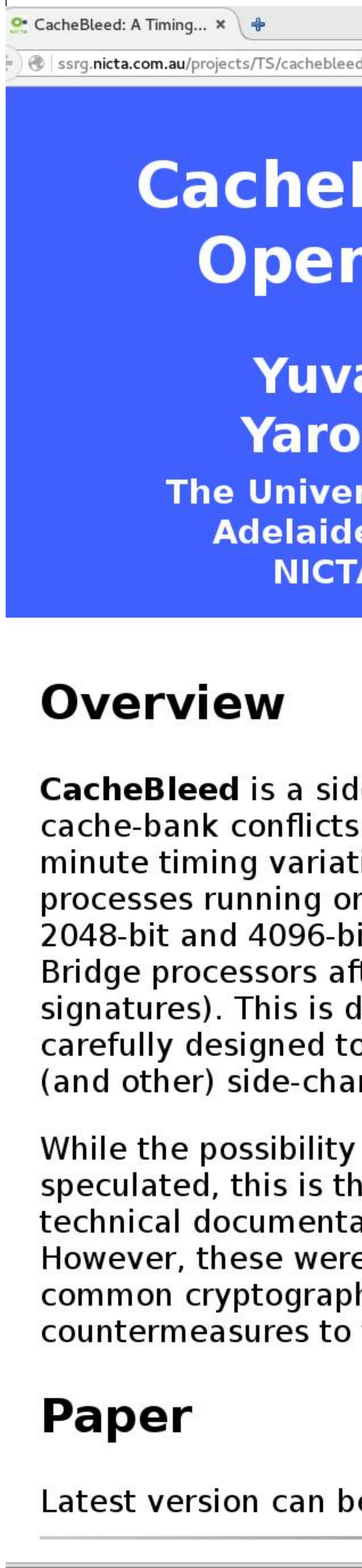

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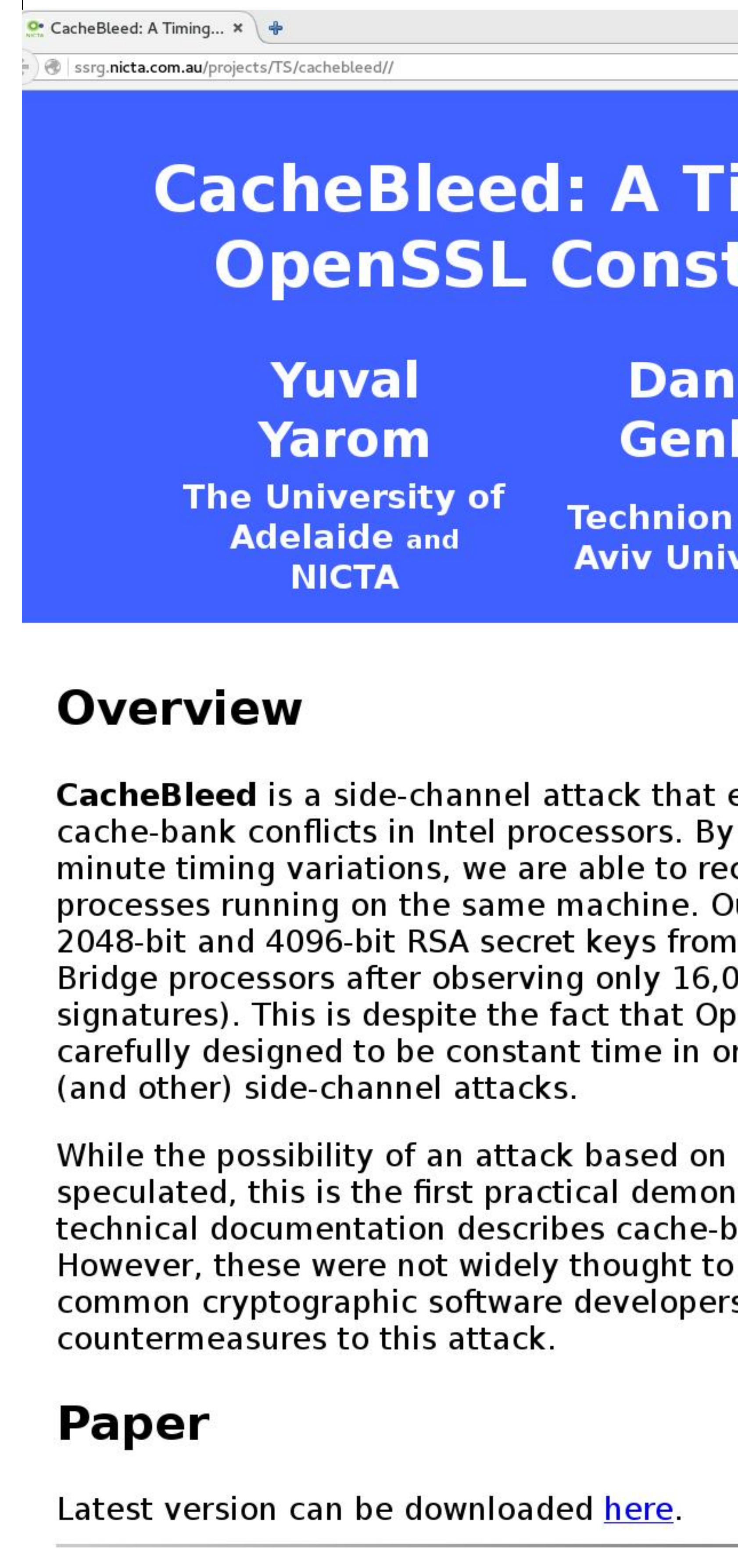

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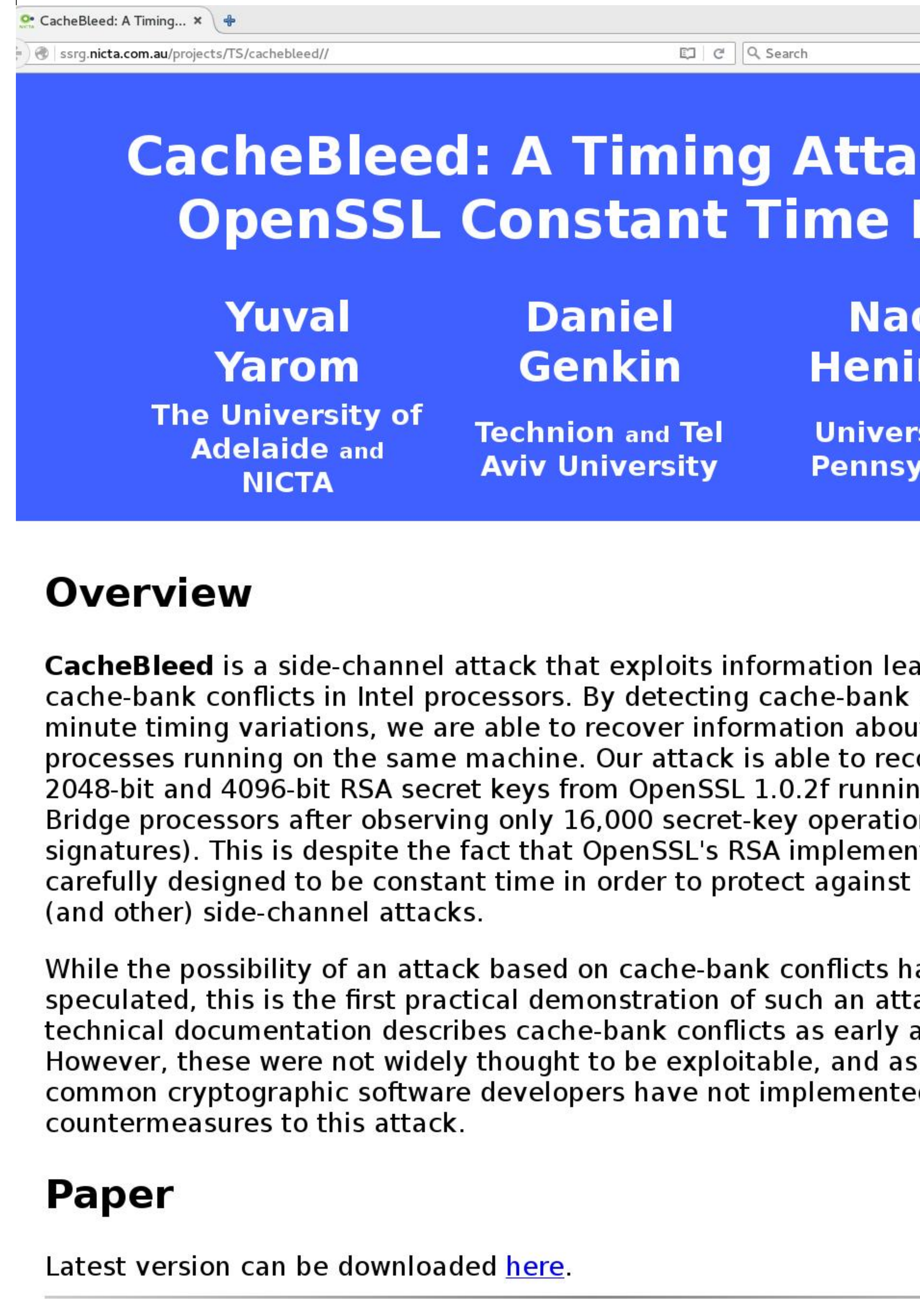

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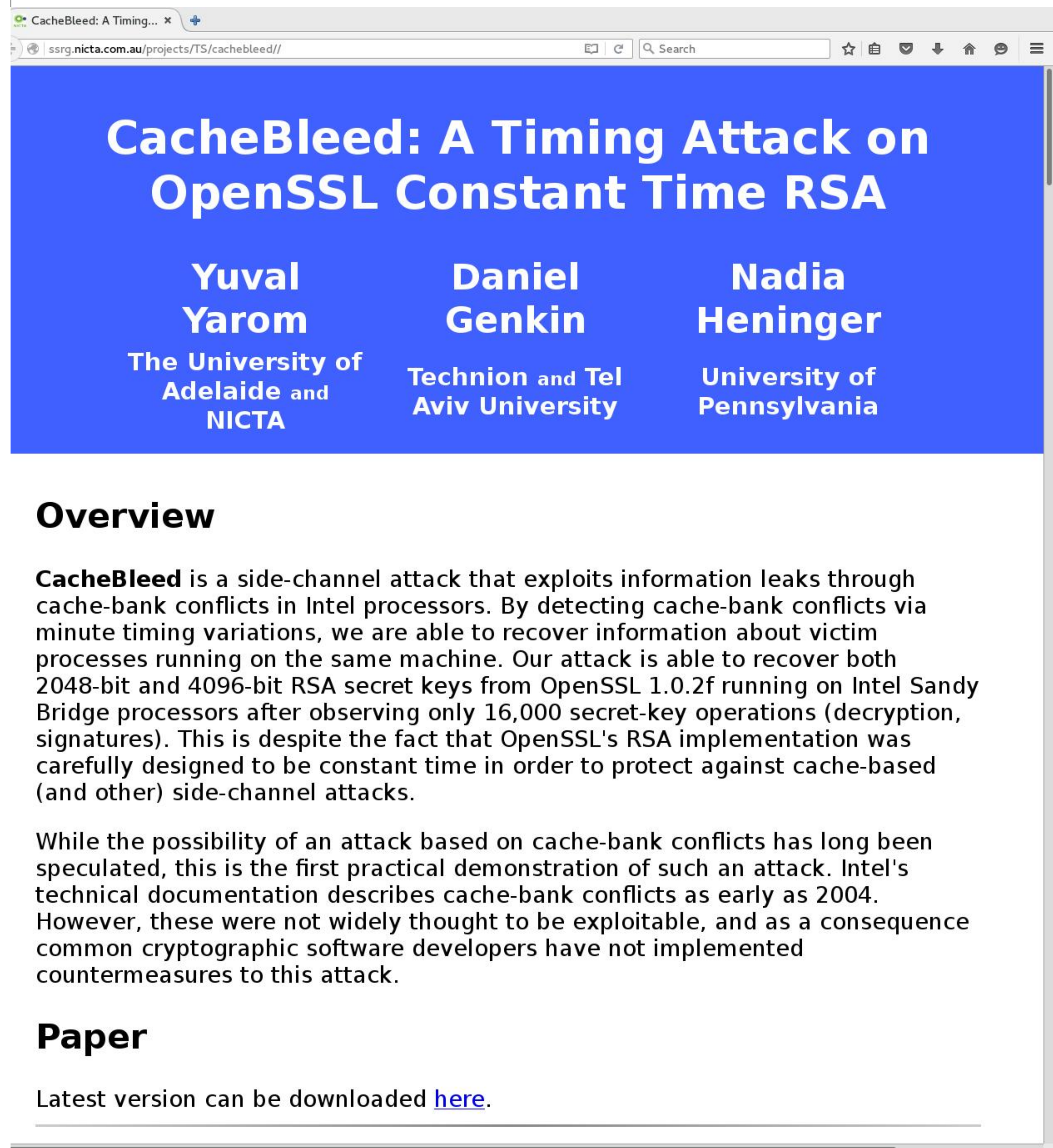
CacheBleed: A Timing...

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# CacheBleed: A Timing Attack on OpenSSL Constant Time RSA

Yuval Yarom — The University of Adelaide and NICTA

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## Overview

**CacheBleed** is a side-channel attack that exploits information leaks through cache-bank conflicts in Intel processors. By detecting cache-bank conflicts via minute timing variations, we are able to recover information about victim processes running on the same machine. Our attack is able to recover both 2048-bit and 4096-bit RSA secret keys from OpenSSL 1.0.2f running on Intel Sandy Bridge processors after observing only 16,000 secret-key operations (decryption, signatures). This is despite the fact that OpenSSL's RSA implementation was carefully designed to be constant time in order to protect against cache-based (and other) side-channel attacks.

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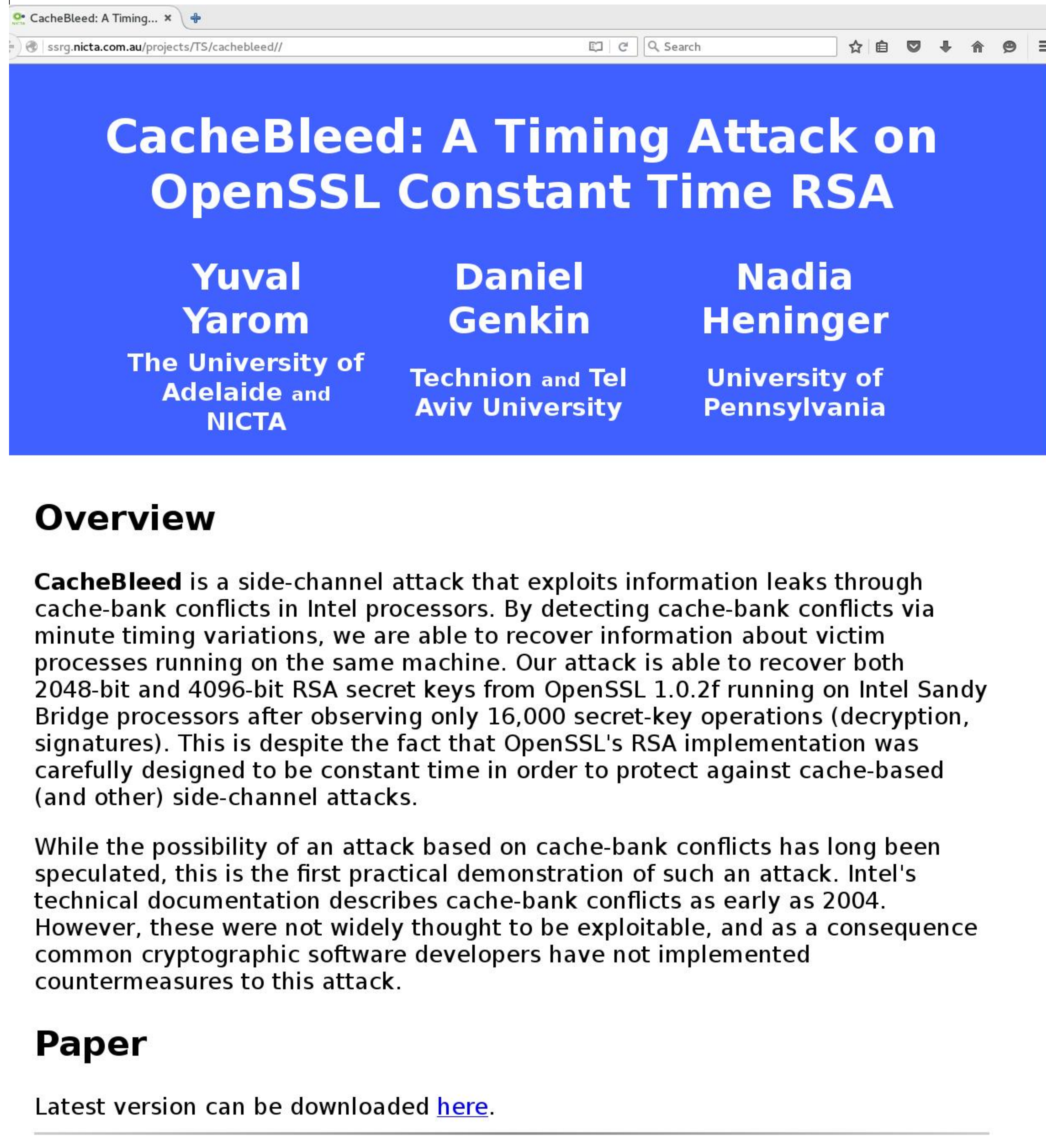


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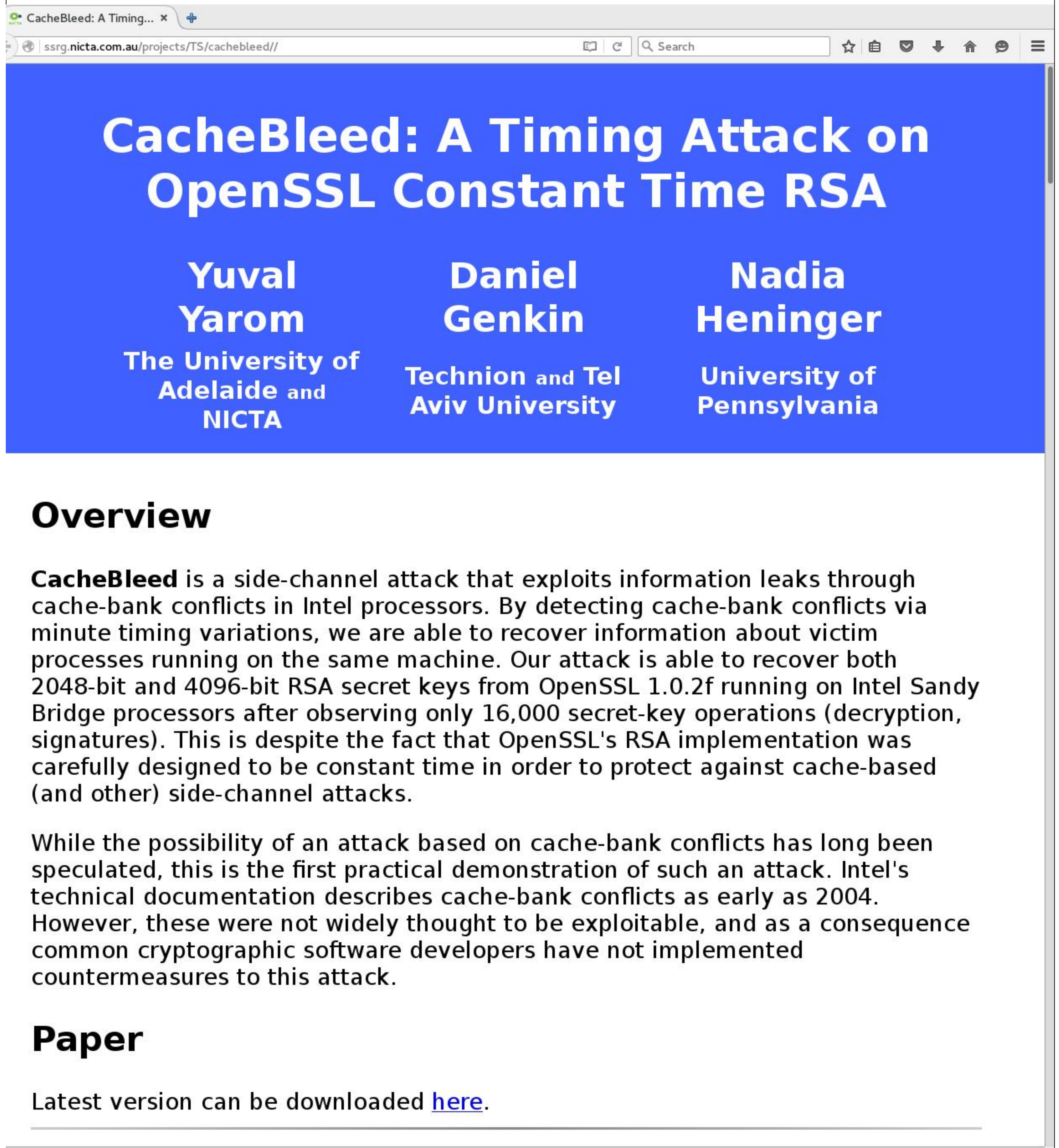


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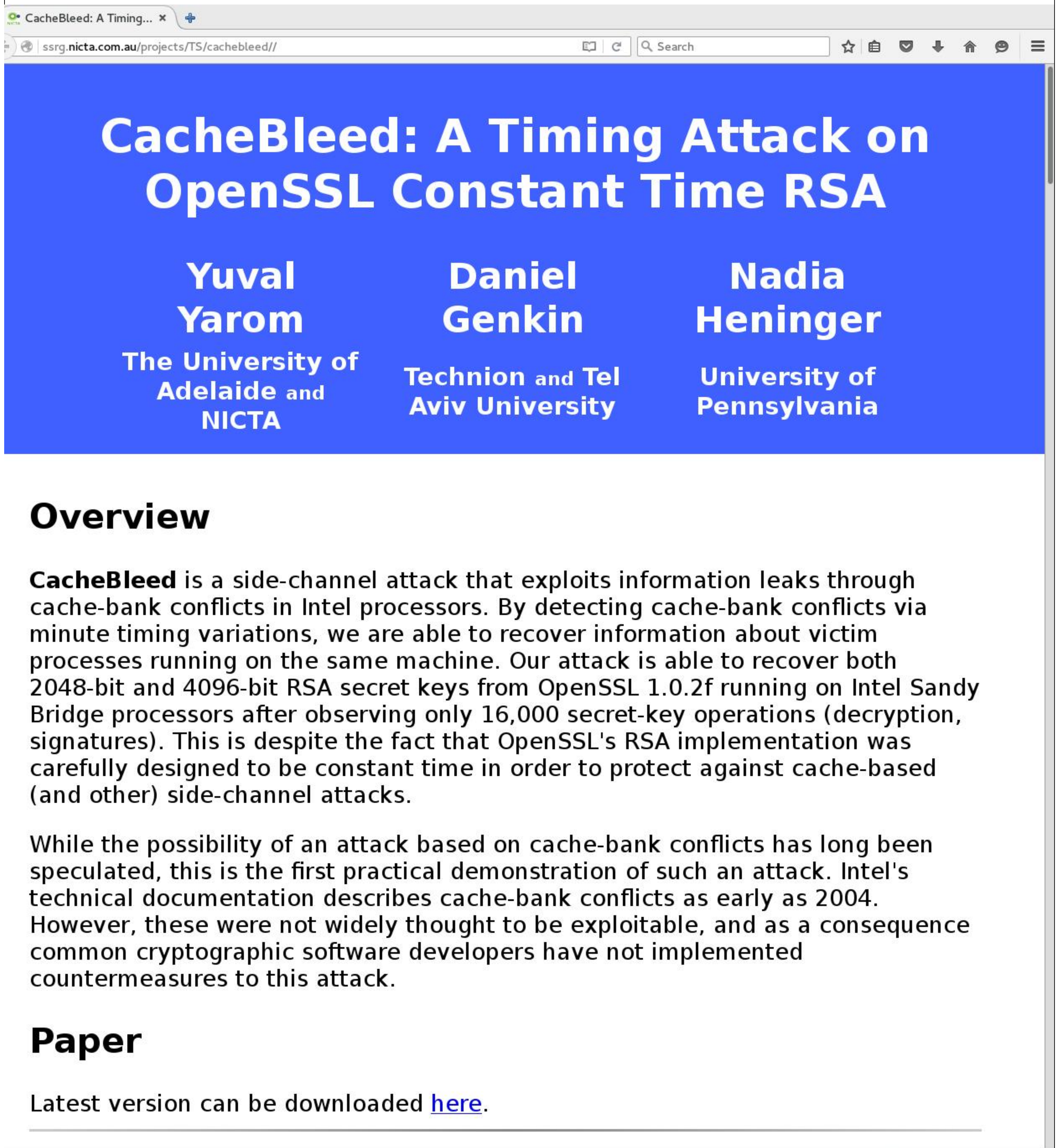


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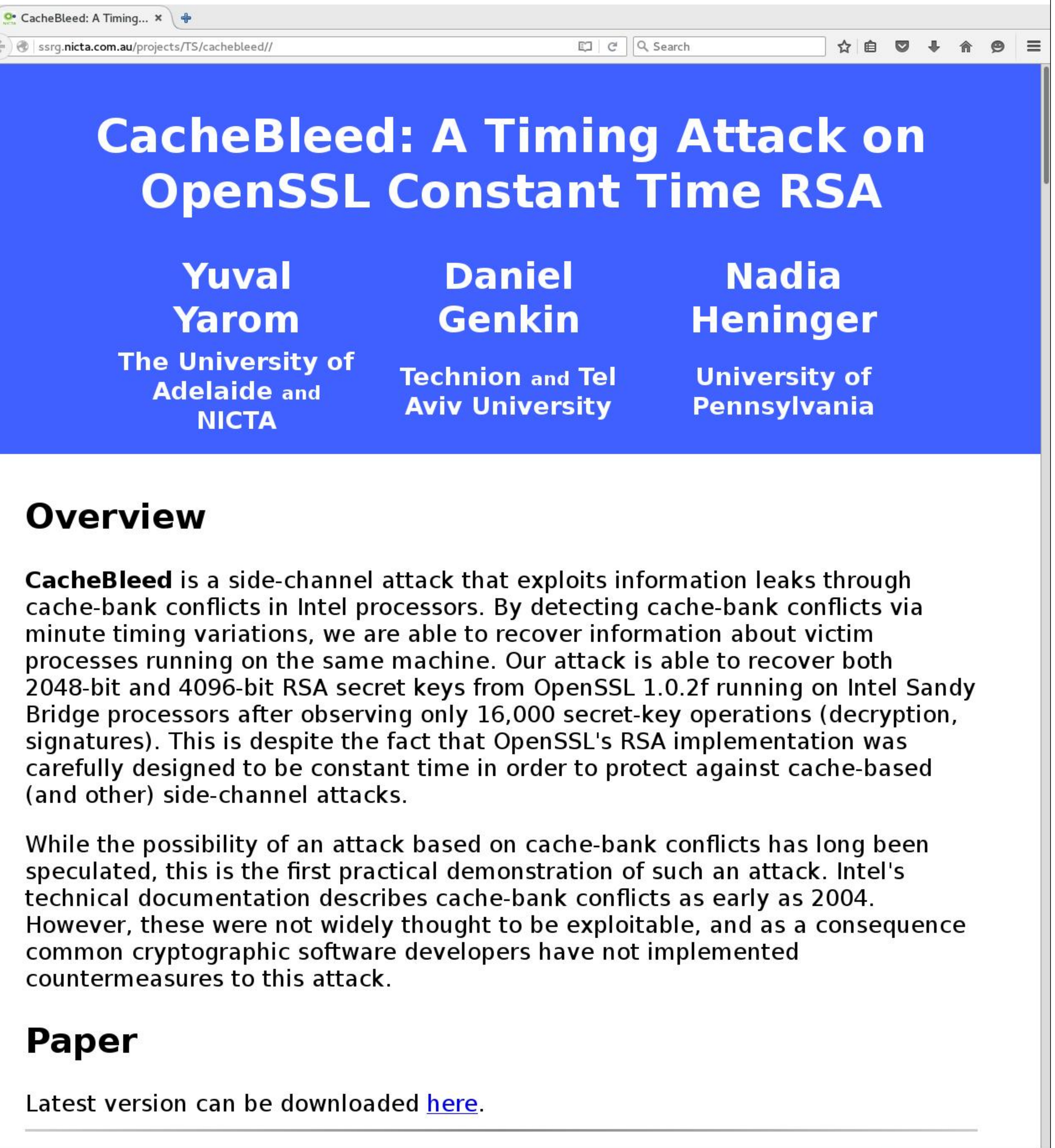




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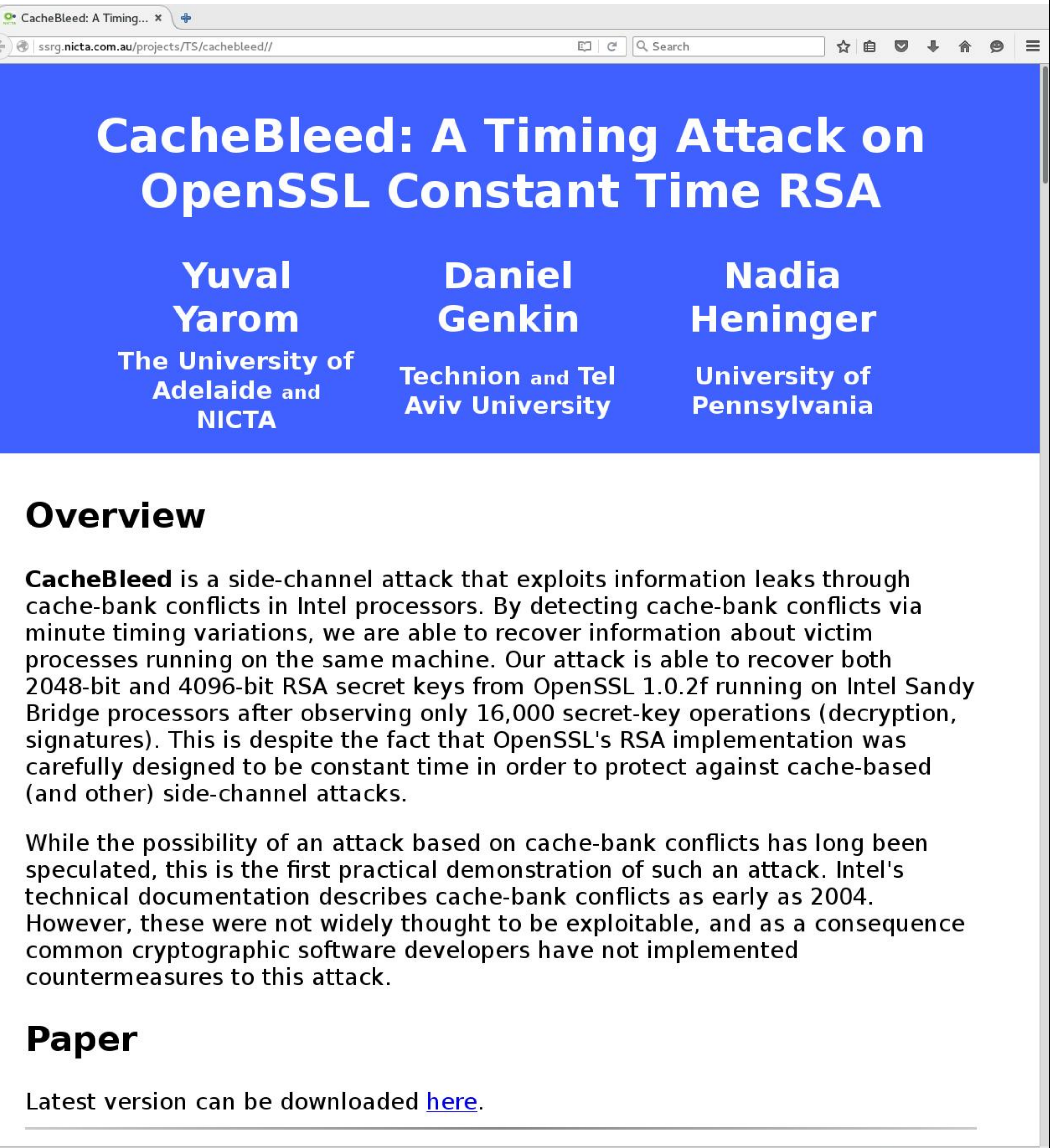

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$\approx$25% of all elliptic curves.

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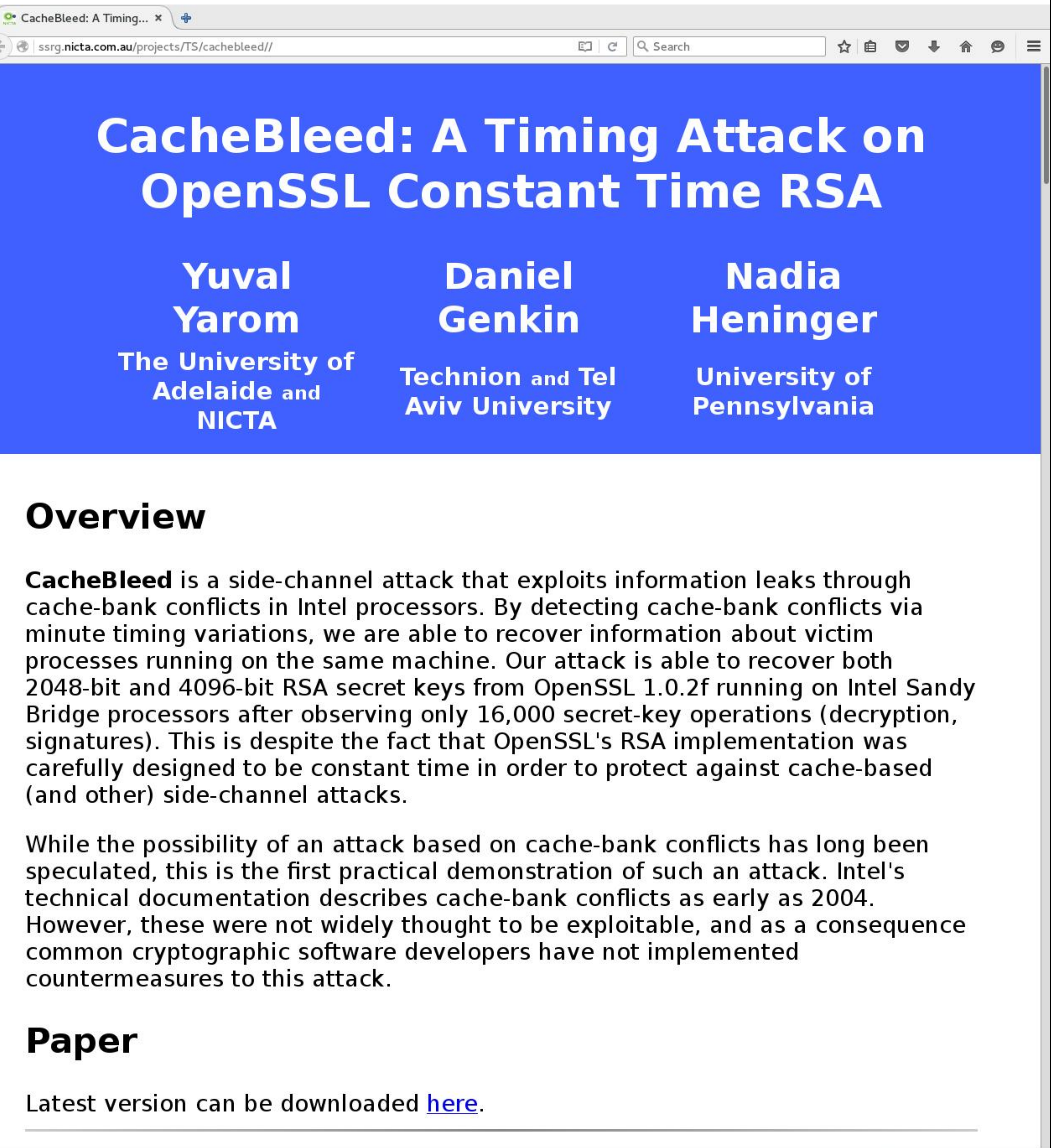

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Choose a curve $y^2 = x^3 + Ax^2 + x$ where $A^2 - 4$ is not a square. $\approx$25% of all elliptic curves.

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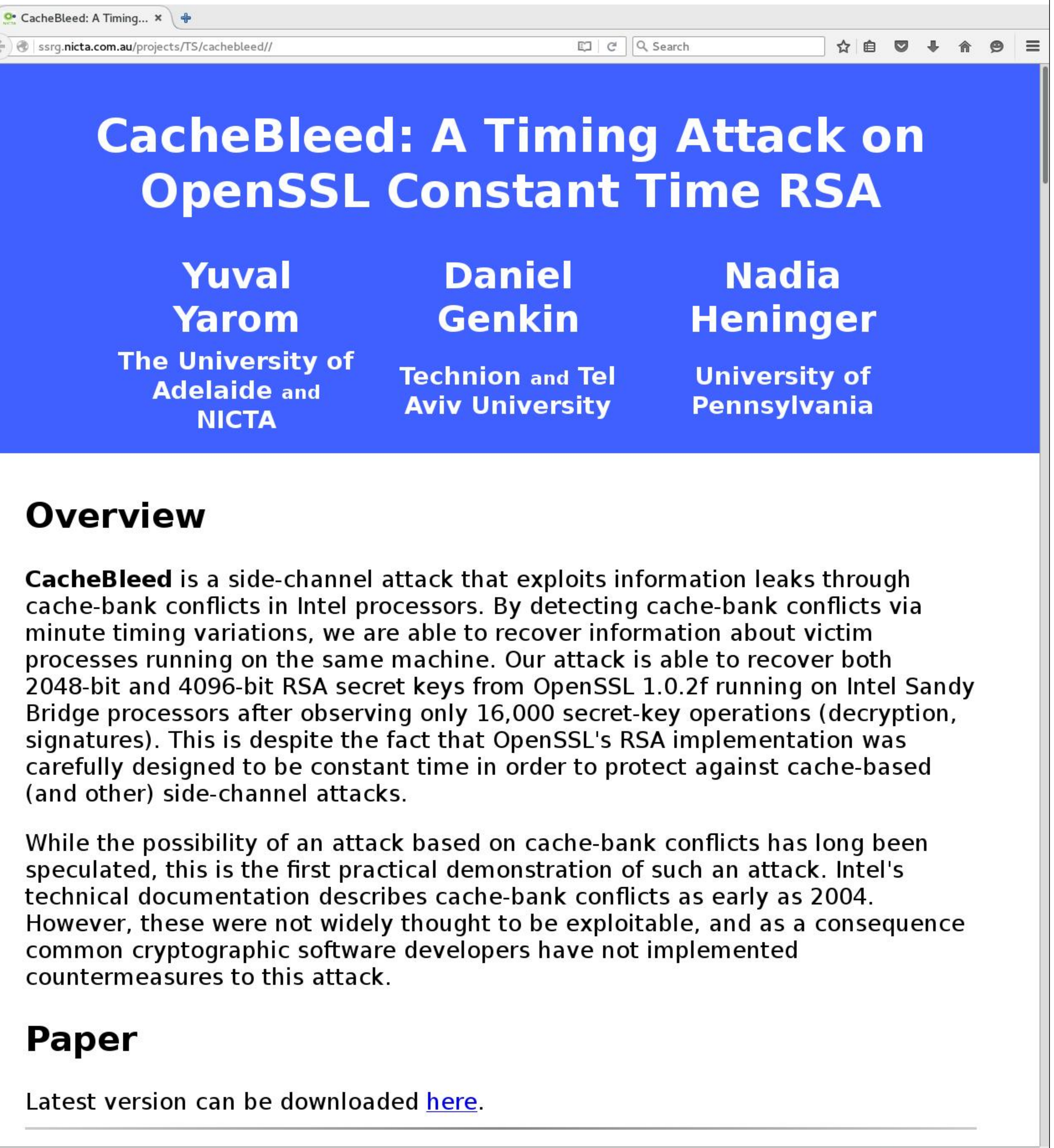
# 2016: OpenSSL didn't listen.




## CacheBleed: A Timing Attack on OpenSSL Constant Time RSA

Yuval Yarom
The University of Adelaide and NICTA

Daniel Genkin
Technion and Tel Aviv University

Nadia Heninger
University of Pennsylvania

### Overview

**CacheBleed** is a side-channel attack that exploits information leaks through cache-bank conflicts in Intel processors. By detecting cache-bank conflicts via minute timing variations, we are able to recover information about victim processes running on the same machine. Our attack is able to recover both 2048-bit and 4096-bit RSA secret keys from OpenSSL 1.0.2f running on Intel Sandy Bridge processors after observing only 16,000 secret-key operations (decryption, signatures). This is despite the fact that OpenSSL's RSA implementation was carefully designed to be constant time in order to protect against cache-based (and other) side-channel attacks.

While the possibility of an attack based on cache-bank conflicts has long been speculated, this is the first practical demonstration of such an attack. Intel's technical documentation describes cache-bank conflicts as early as 2004. However, these were not widely thought to be exploitable, and as a consequence common cryptographic software developers have not implemented countermeasures to this attack.

### Paper

Latest version can be downloaded here.



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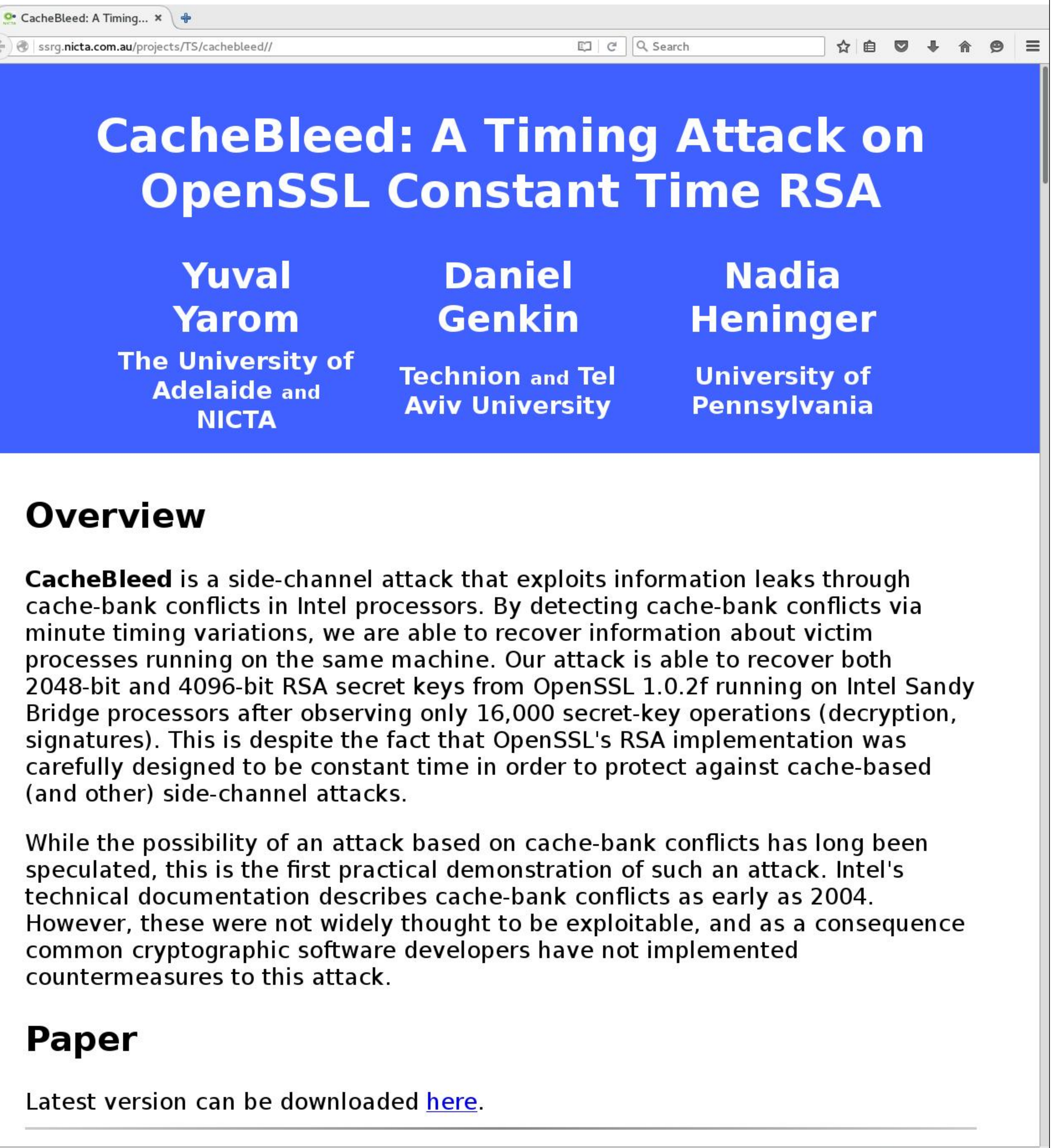


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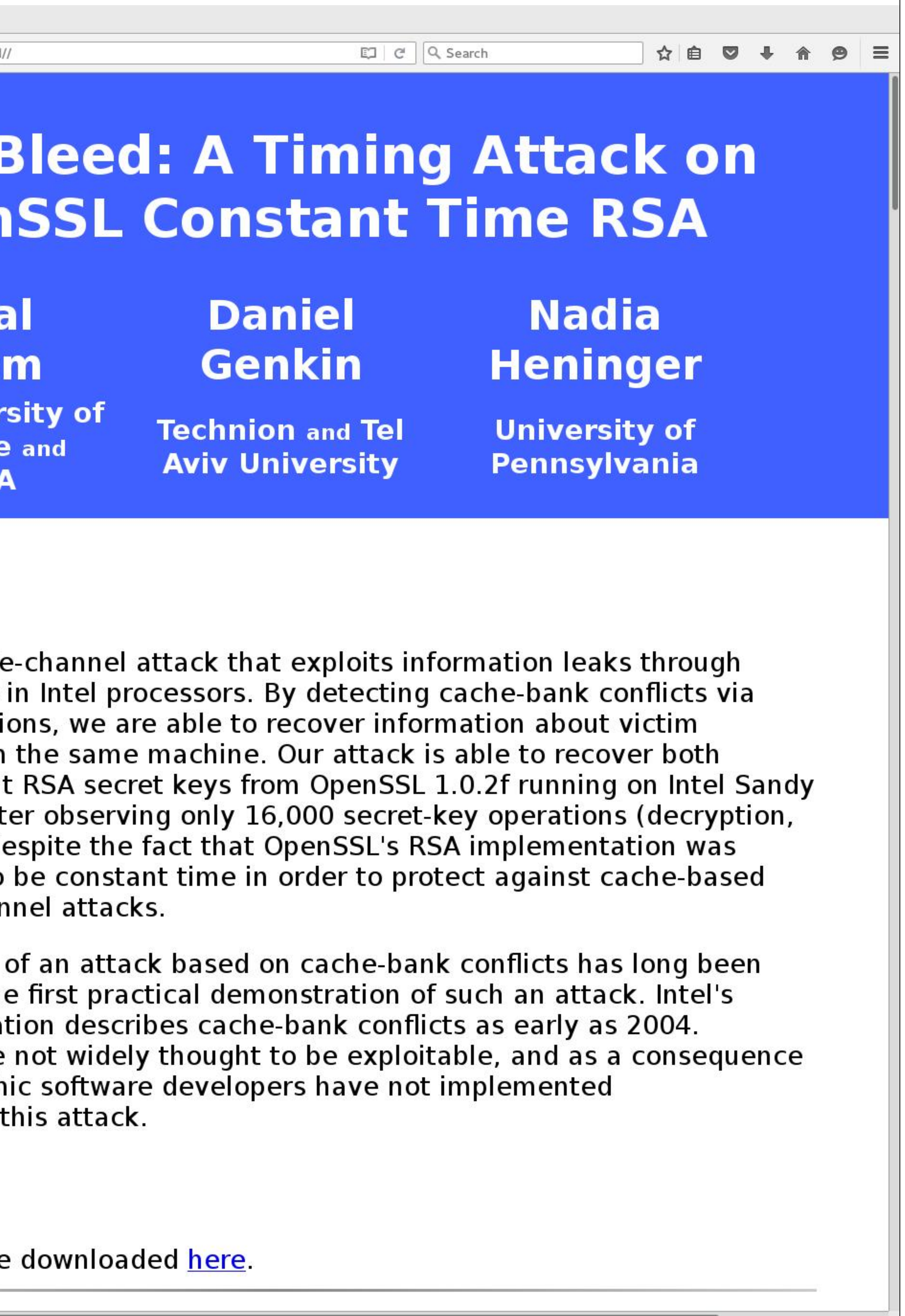




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```
x2,z2,x3
for i in
  bit =
  x2,x3
  z2,z3
  x3,z3


  x2,z2
    4*x2
  x2,x3
  z2,z3
return x
```

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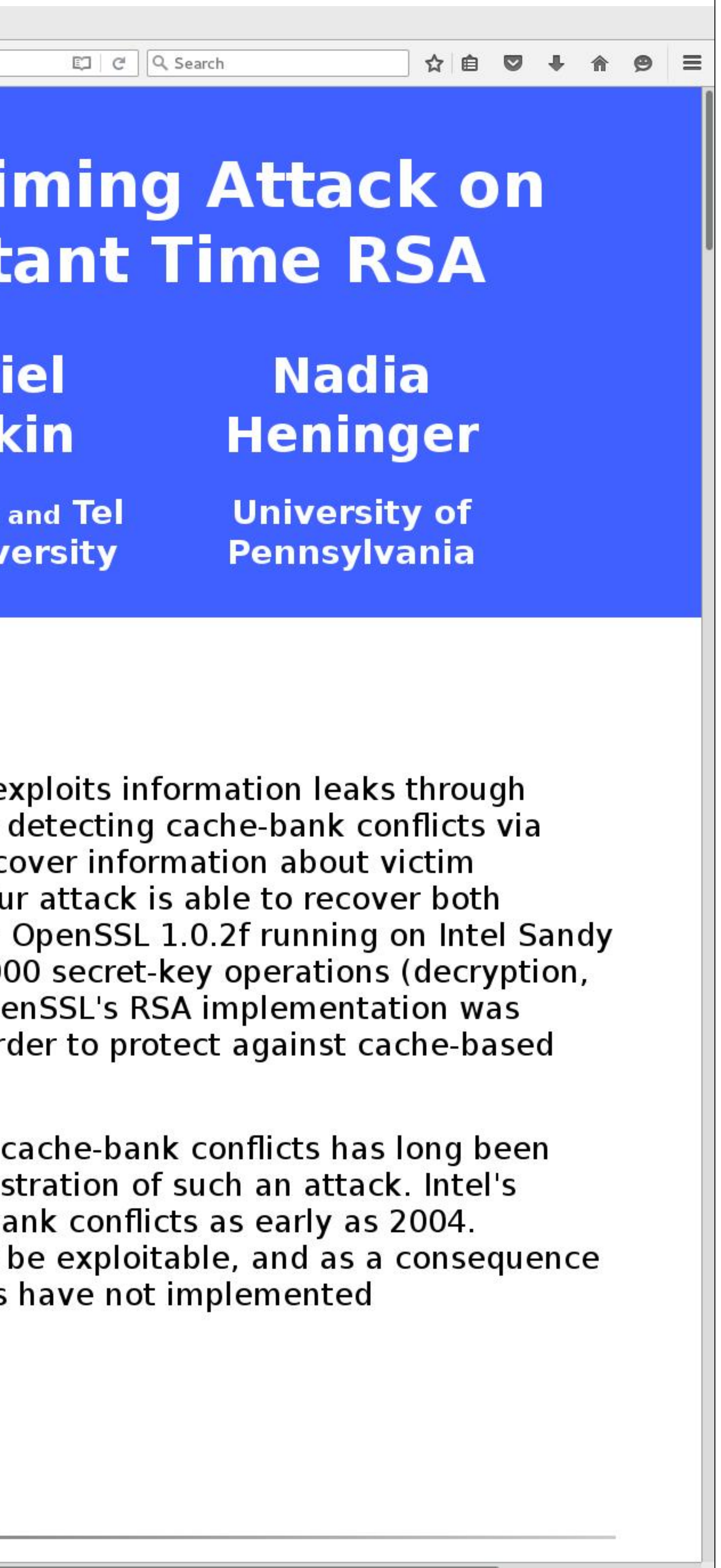




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```
x2,z2,x3,z3 = 1,
for i in reverse
  bit = 1 & (n >
  x2,x3 = cswap(
  z2,z3 = cswap(
  x3,z3 = ((x2*x
        x1*(x2*z
  x2,z2 = ((x2^2
    4*x2*z2*(x2^
  x2,x3 = cswap(
  z2,z3 = cswap(
return x2*z2^(p-
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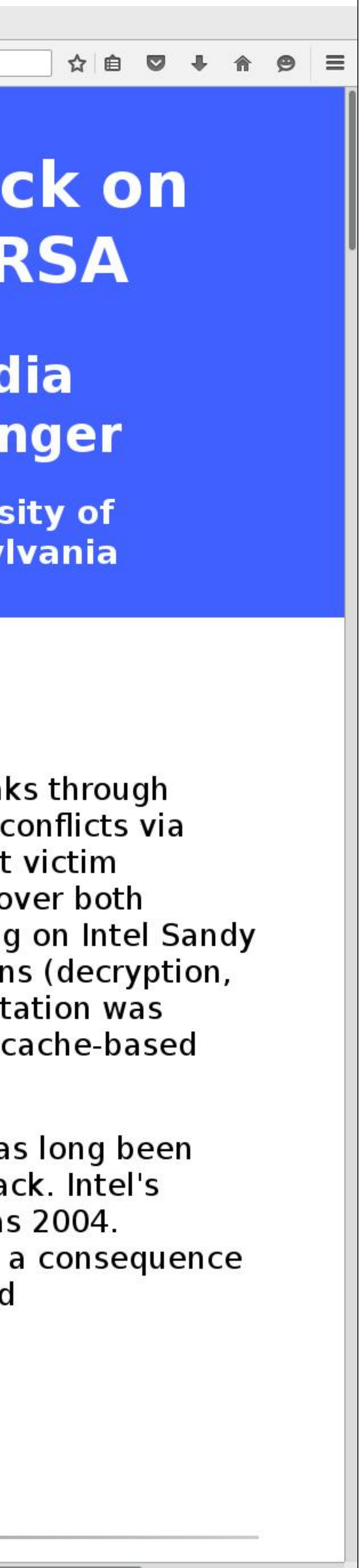



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```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(2
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit
  z2,z3 = cswap(z2,z3,bit
  x3,z3 = ((x2*x3-z2*z3)^
        x1*(x2*z3-z2*x3)^
  x2,z2 = ((x2^2-z2^2)^2,
    4*x2*z2*(x2^2+A*x2*z2
  x2,x3 = cswap(x2,x3,bit
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return x2*z2^(p-2)
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```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
  x3,z3 = ((x2*x3-z2*z3)^2,
           x1*(x2*z3-z2*x3)^2)
  x2,z2 = ((x2^2-z2^2)^2,
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Curve25519: Change initialization to allow leading 0 bits.
Use constant #loops.

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Use arithmetic to compute `cswap` in constant time.

```
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n reversed(range(255)):
 1 & (n >> i)
 = cswap(x2,x3,bit)
 = cswap(z2,z3,bit)
 = ((x2*x3-z2*z3)^2,
 x1*(x2*z3-z2*x3)^2)
 = ((x2^2-z2^2)^2,
2*z2*(x2^2+A*x2*z2+z2^2))
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 = cswap(z2,z3,bit)
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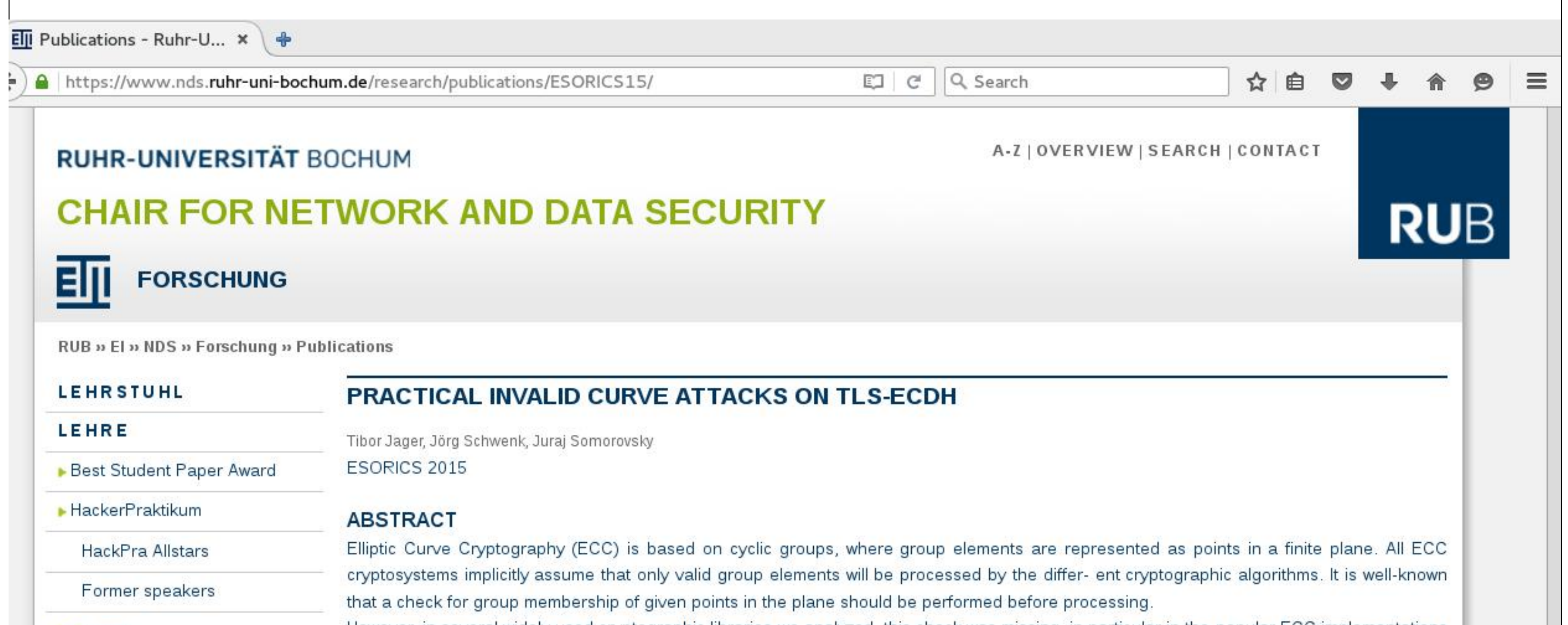


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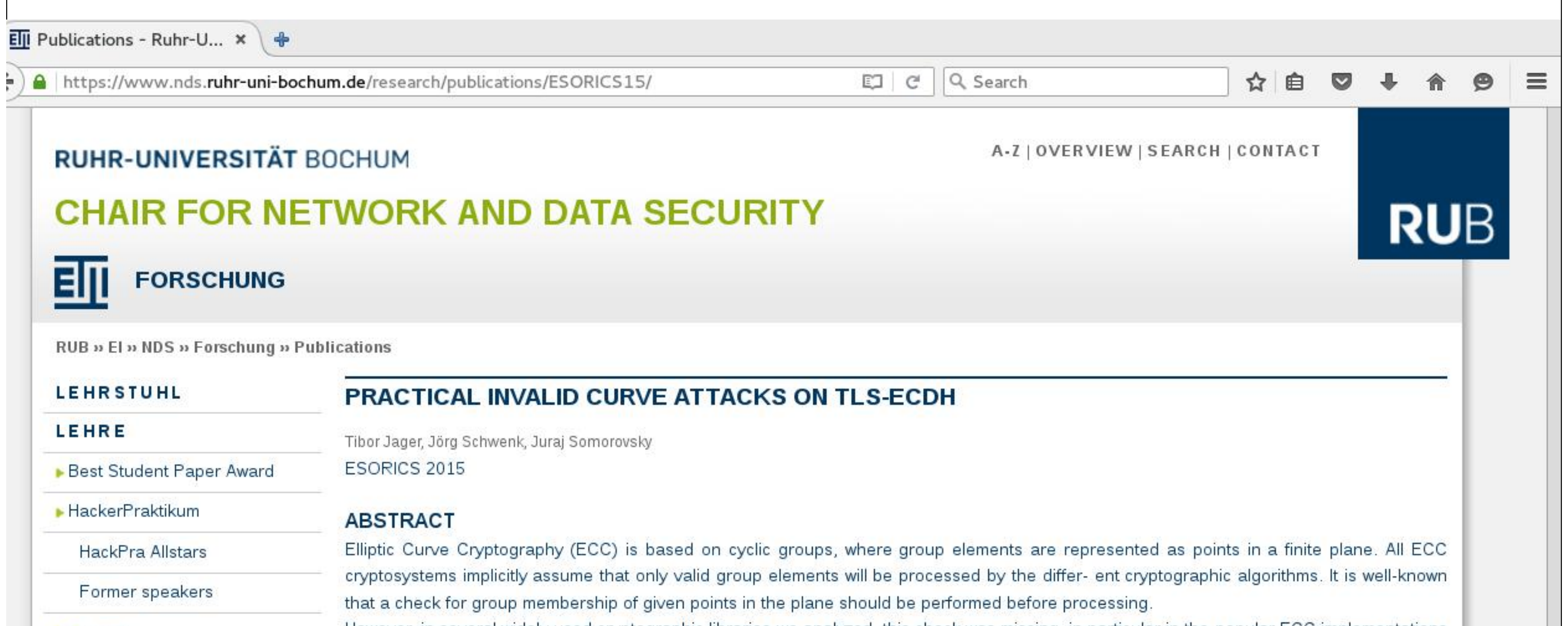


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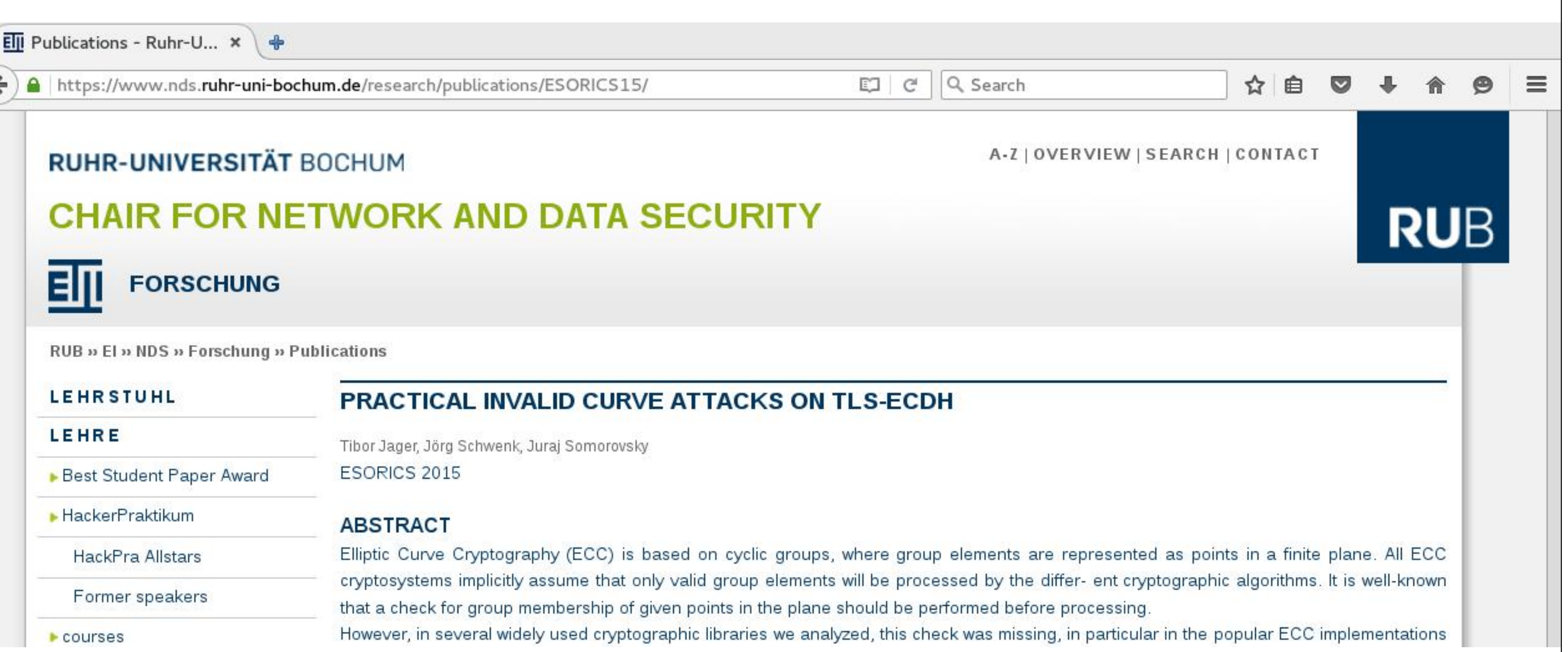
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Best Student Paper Award
HackerPraktikum
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PRACTICAL INVALID CURVE ATTACKS ON TLS-ECDH

Tibor Jager, Jörg Schwenk, Juraj Somorovsky

ESORICS 2015

ABSTRACT

Elliptic Curve Cryptography (ECC) is based on cyclic groups, where group elements are represented as points in a finite plane. All ECC cryptosystems implicitly assume that only valid group elements will be processed by the differ- ent cryptographic algorithms. It is well-known that a check for group membership of given points in the plane should be performed before processing.

However, in several widely used cryptographic libraries we analyzed, this check was missing, in particular in the popular ECC implementations



Curve25519 paper: "free key validation" eliminates these attacks. No cost for checking input; no code to forget.

1. Montgomery naturally follows 1986 Miller compression: send only $x$-coordinate, not $(x, y)$. Forces input onto "curve" or "twist". (Bonus: 32-byte keys!)

2. Montgomery ladder works correctly for inputs on twist.

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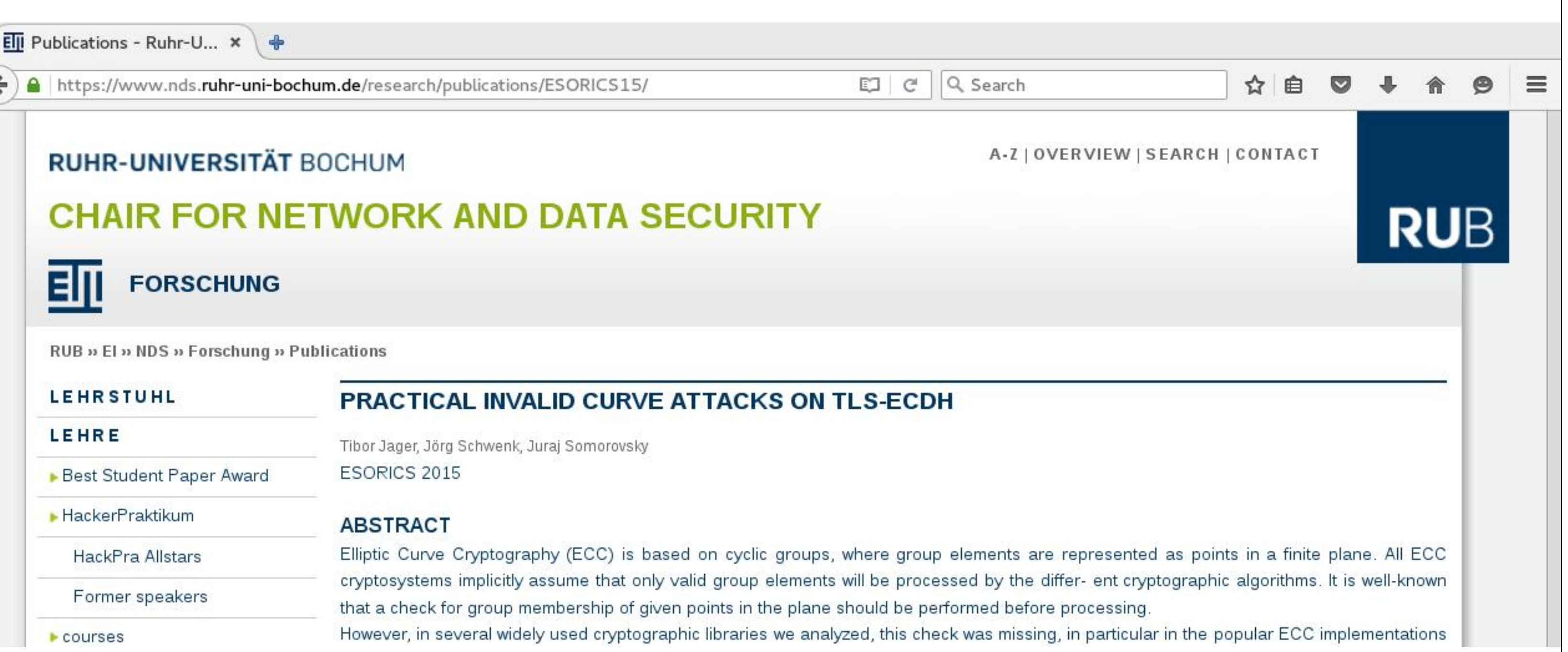




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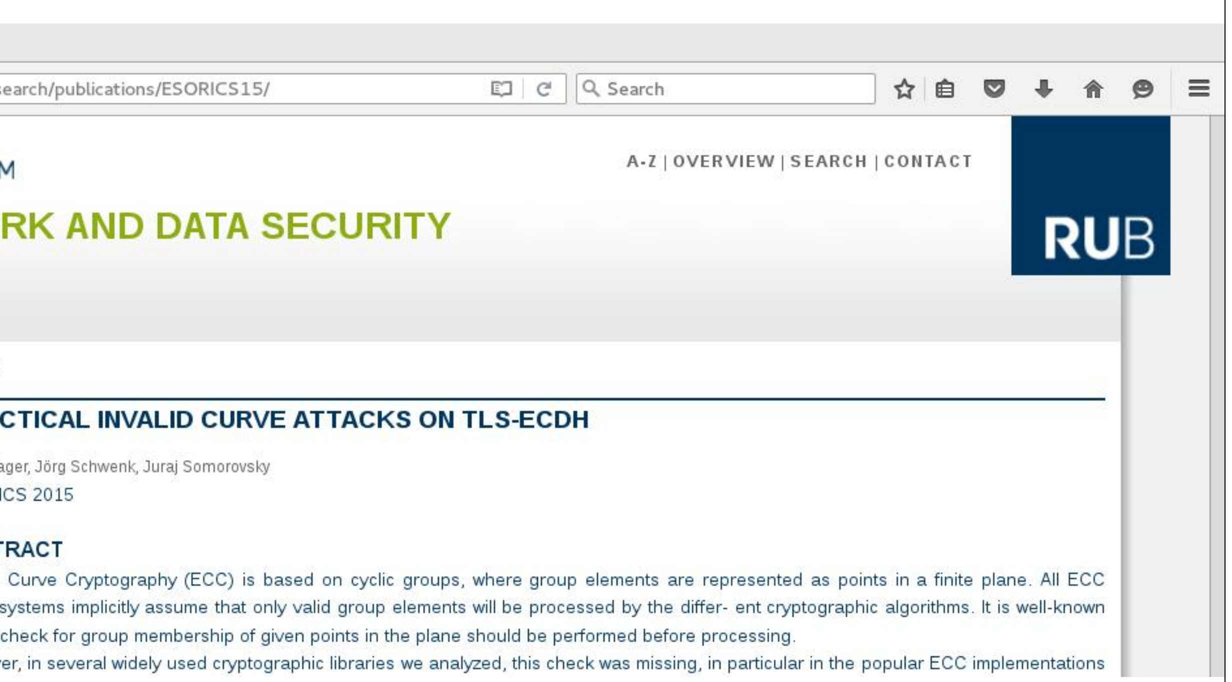




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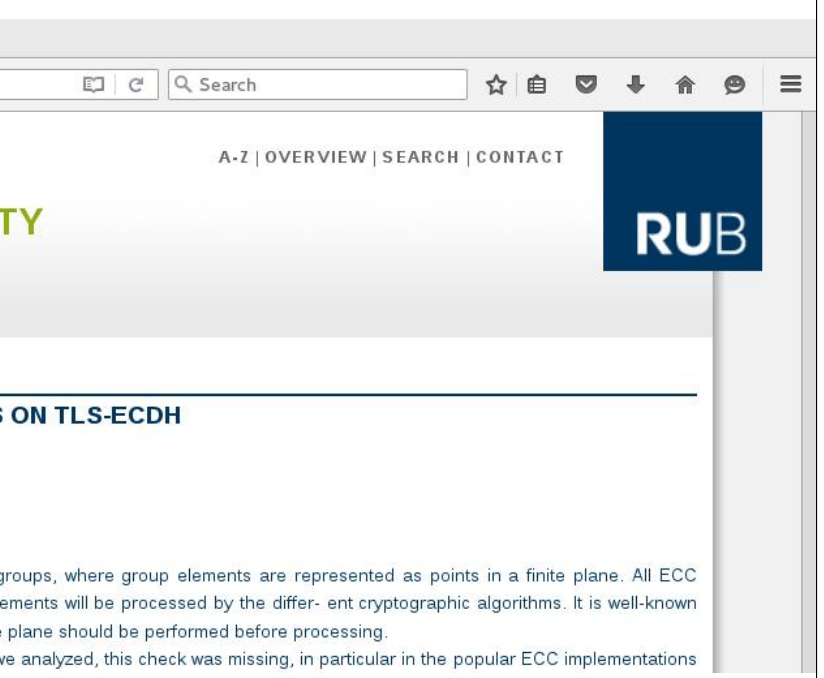




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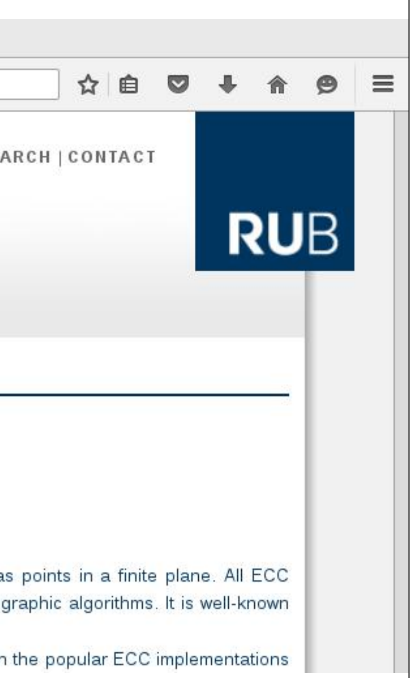




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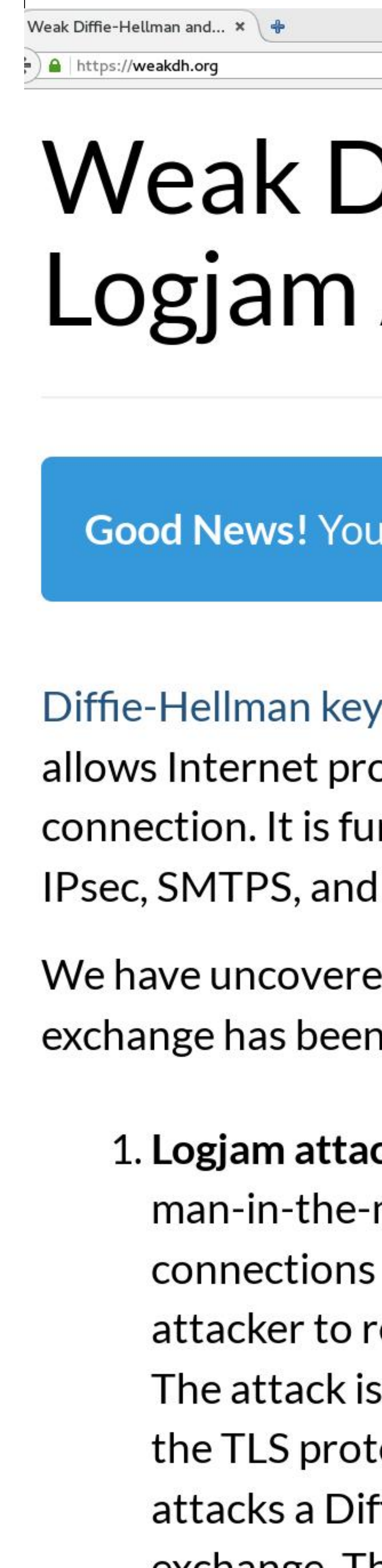

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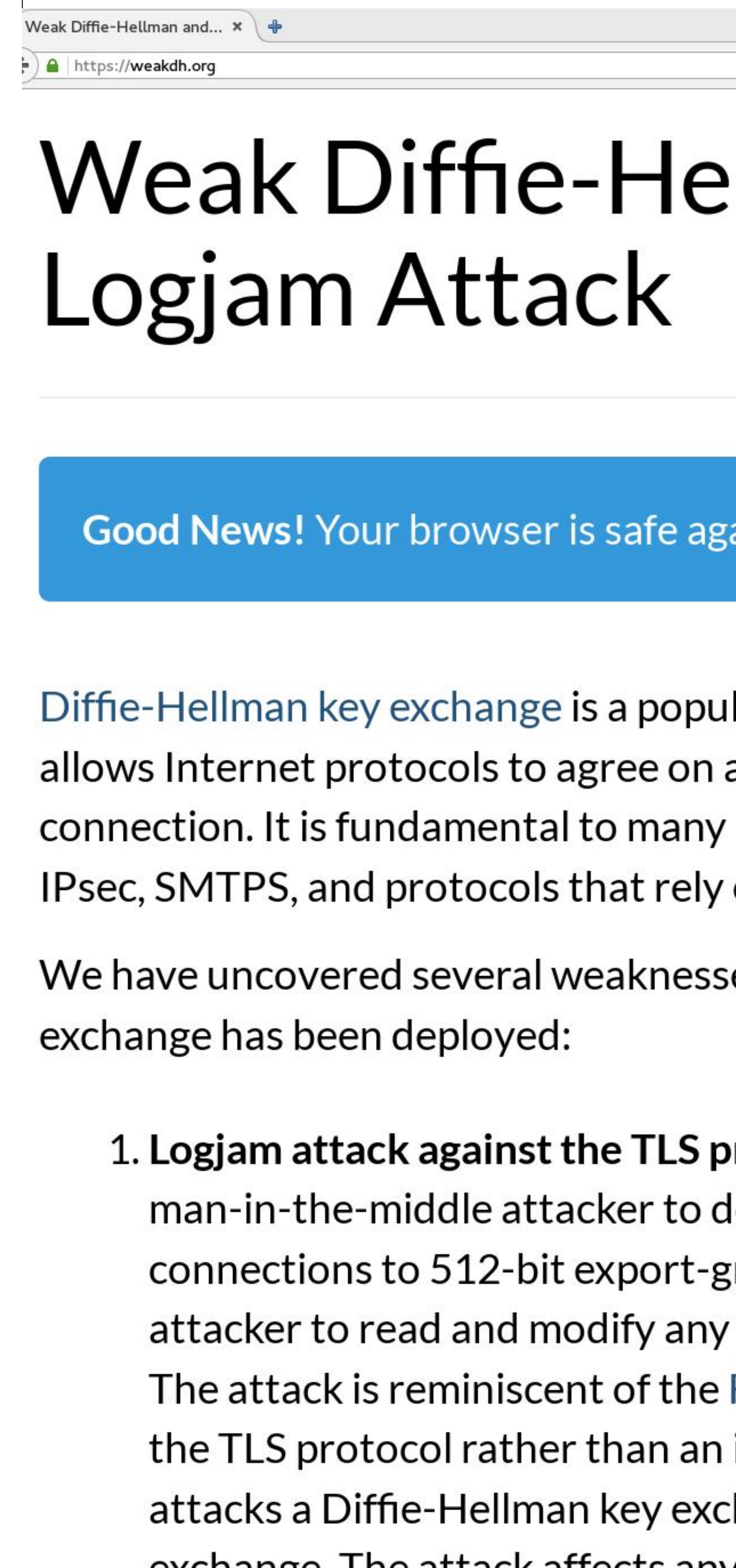

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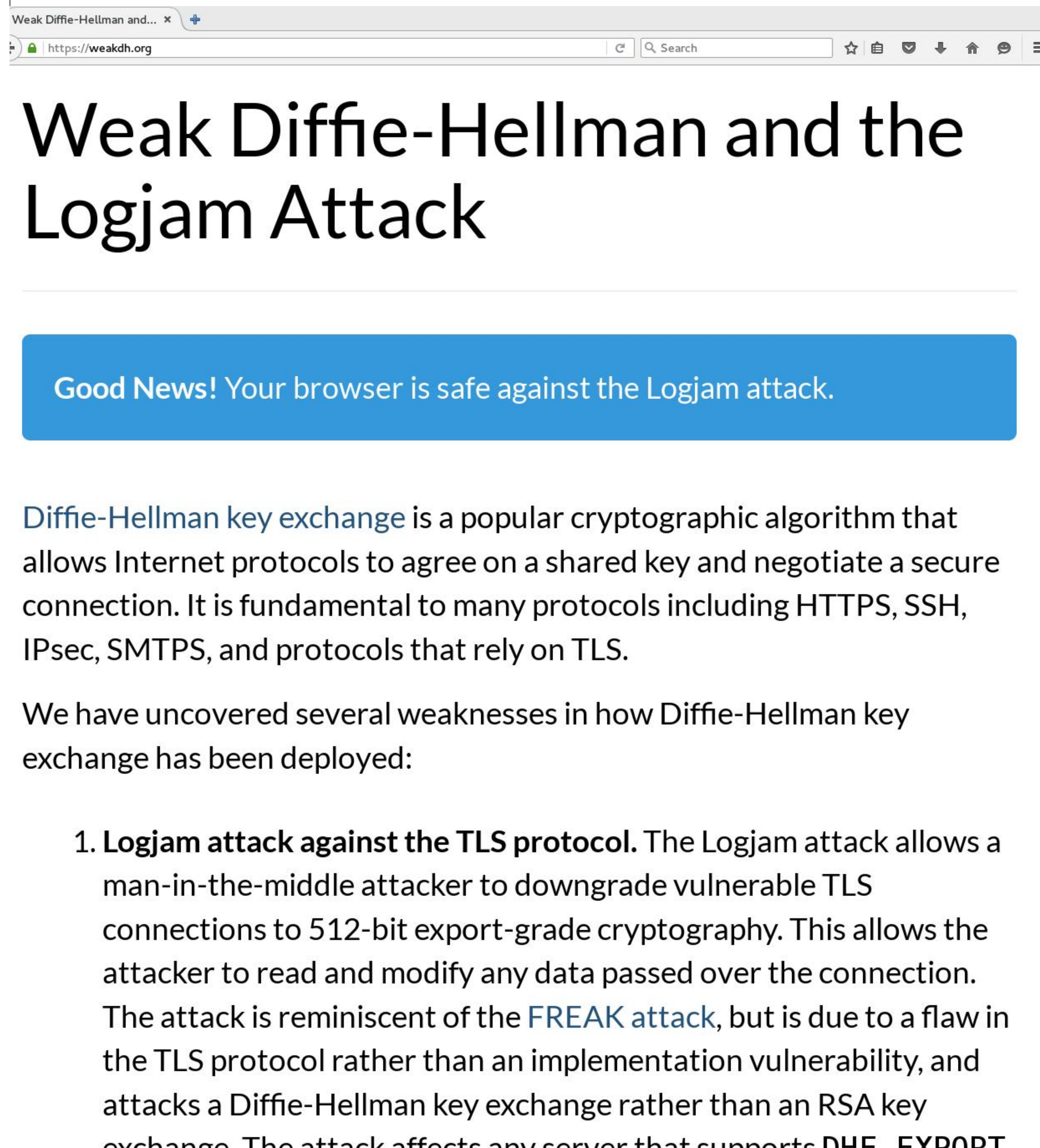
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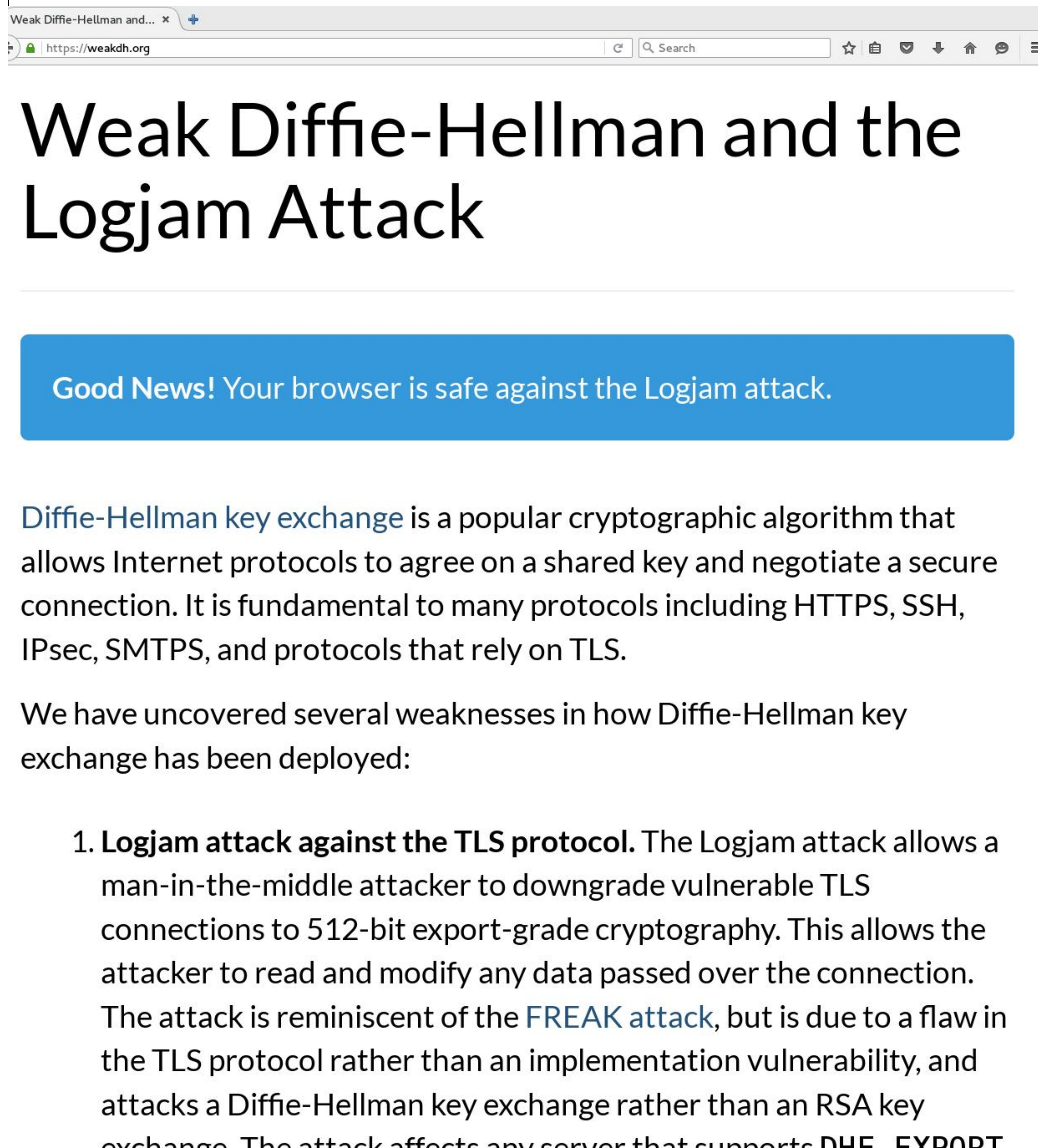
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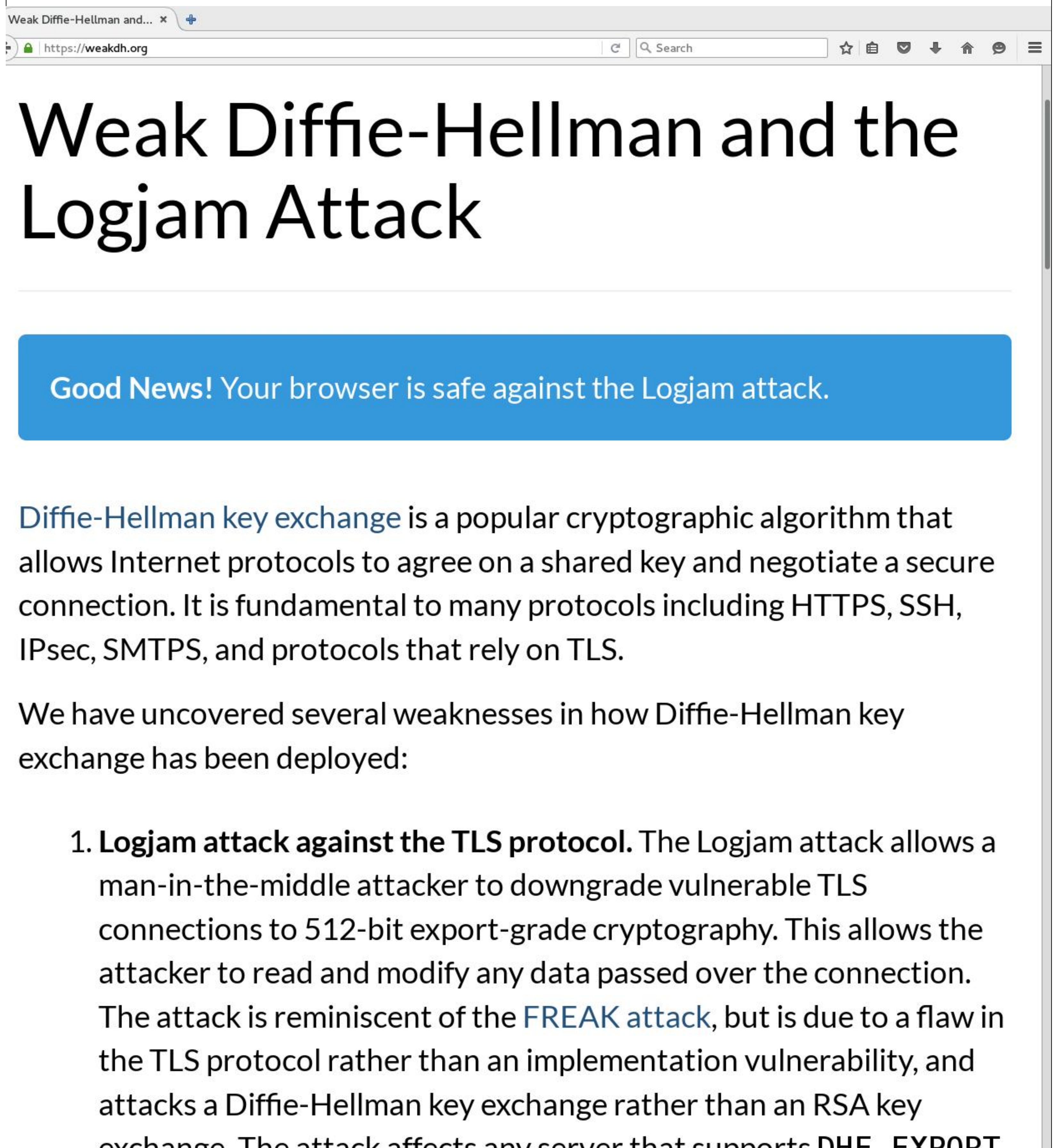


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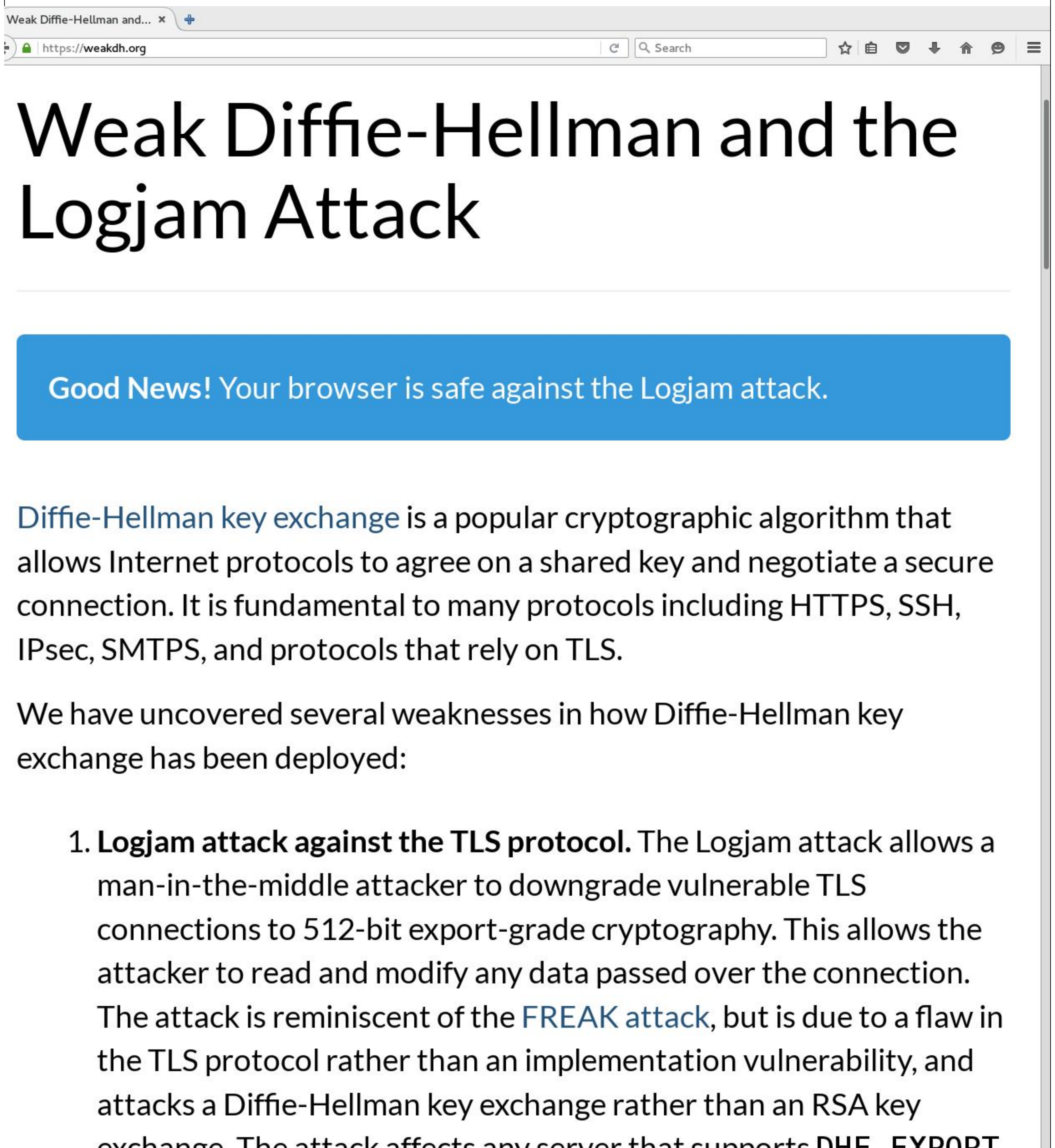
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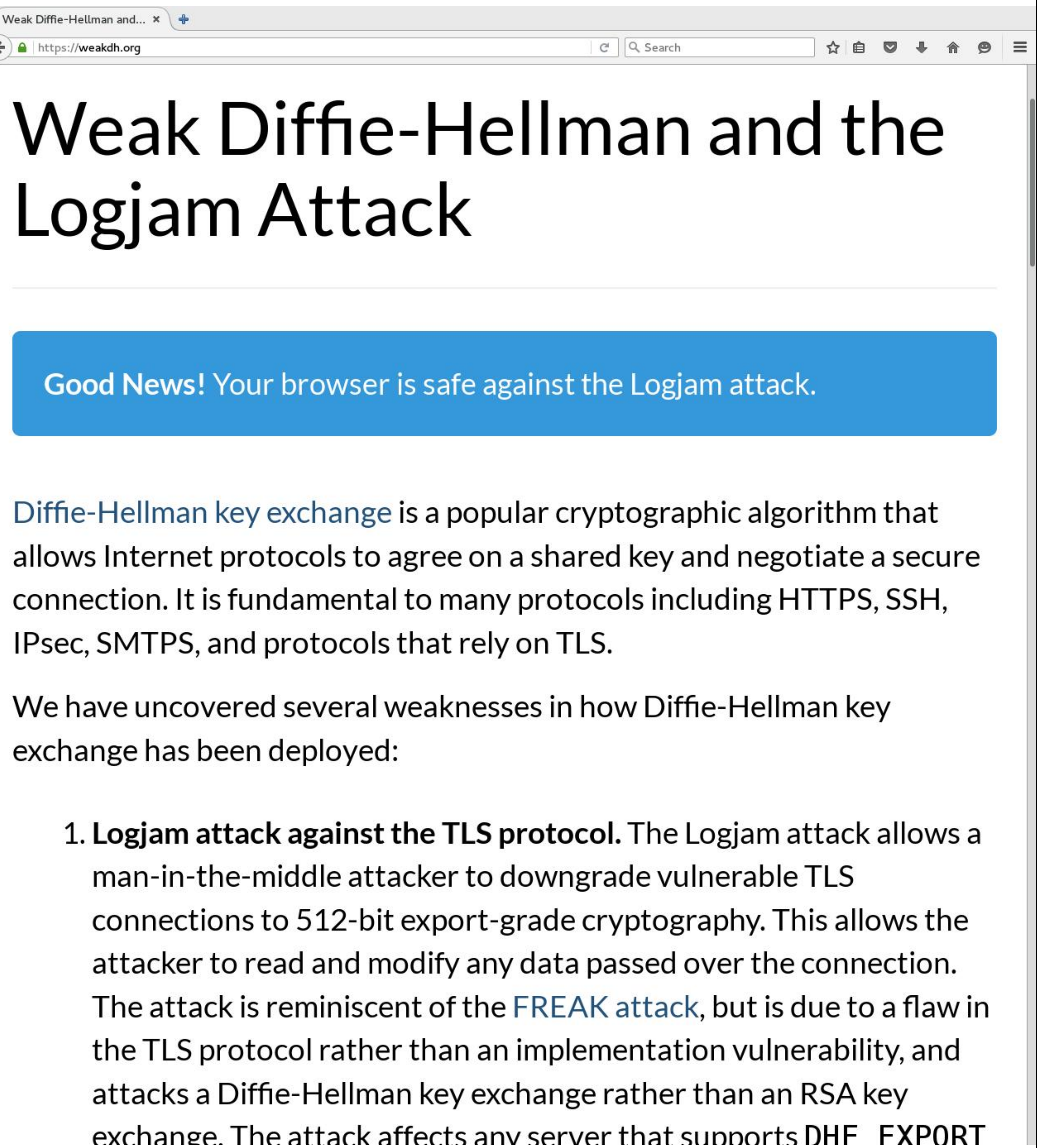
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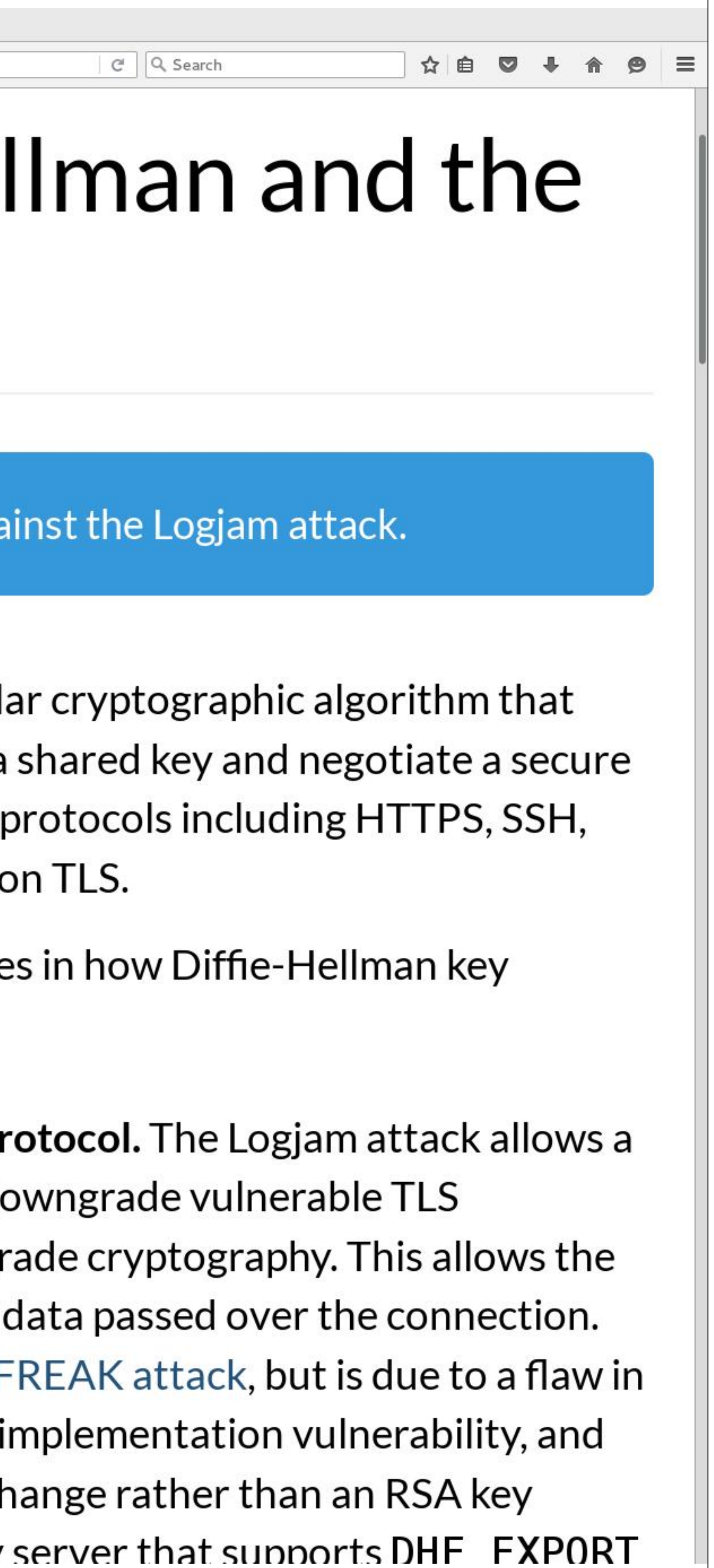




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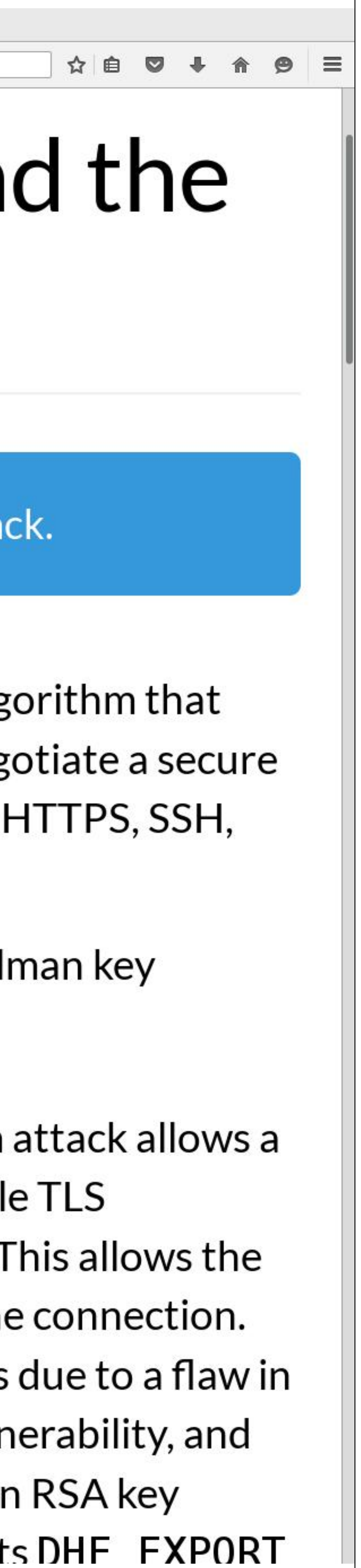




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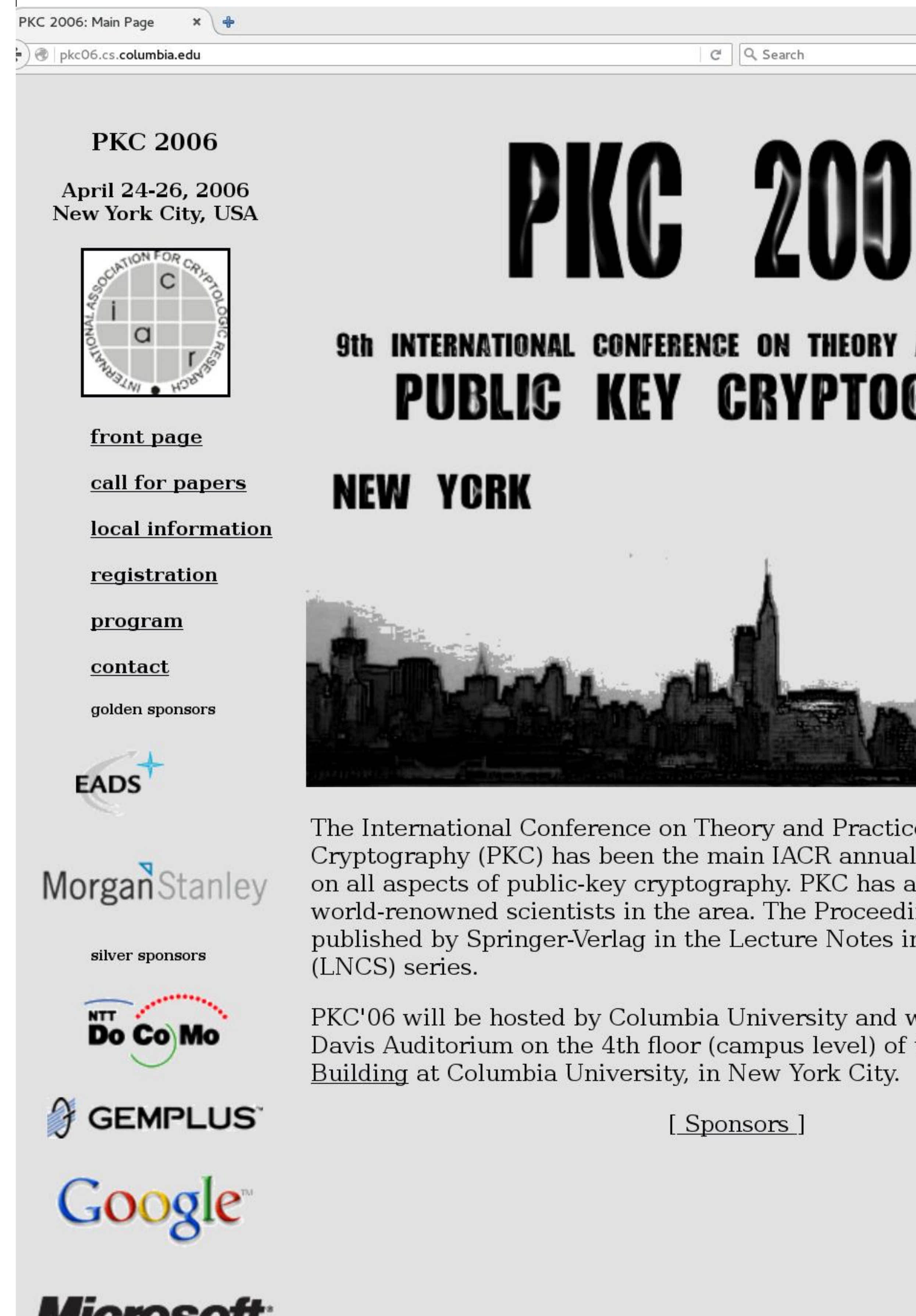

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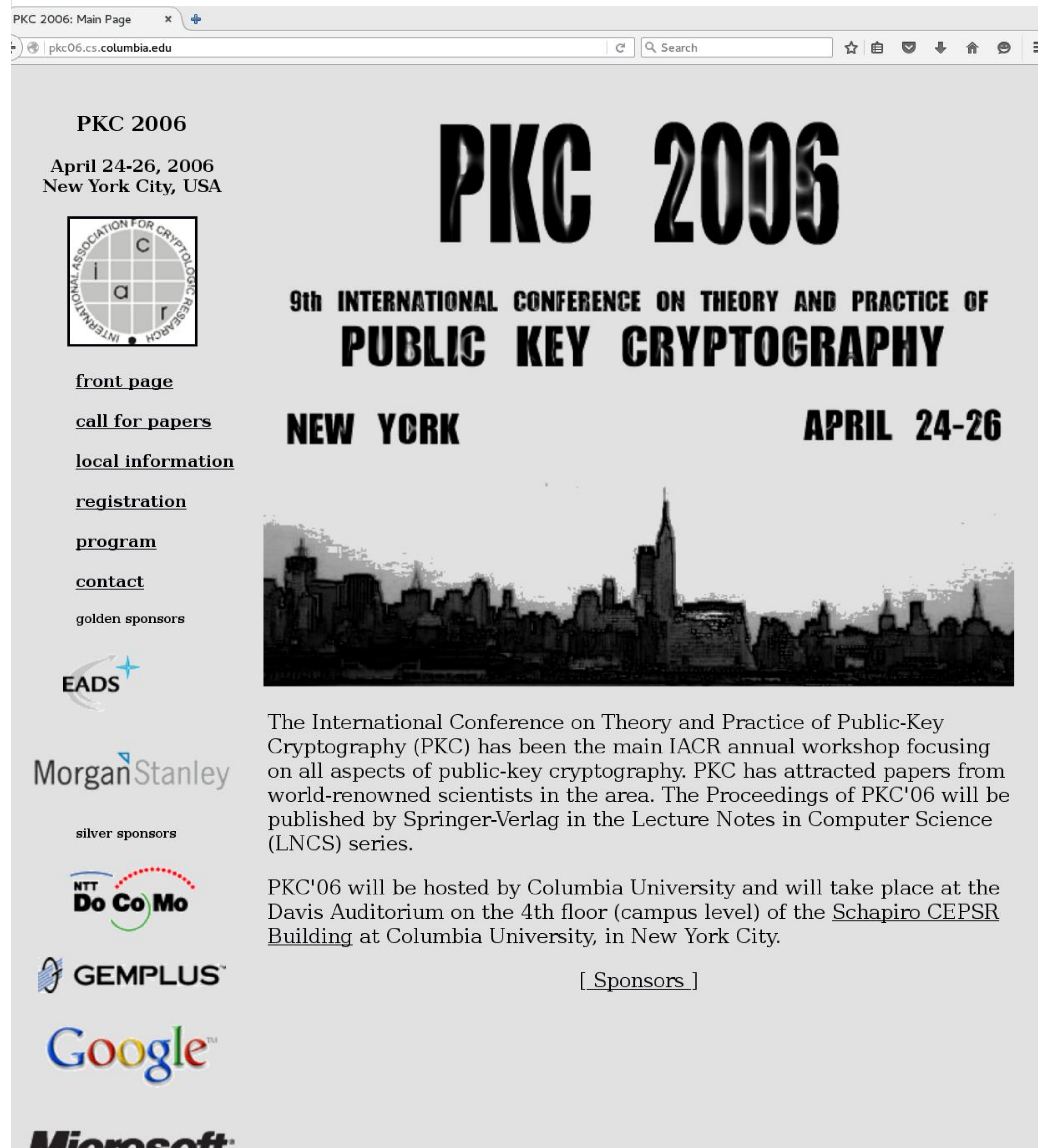

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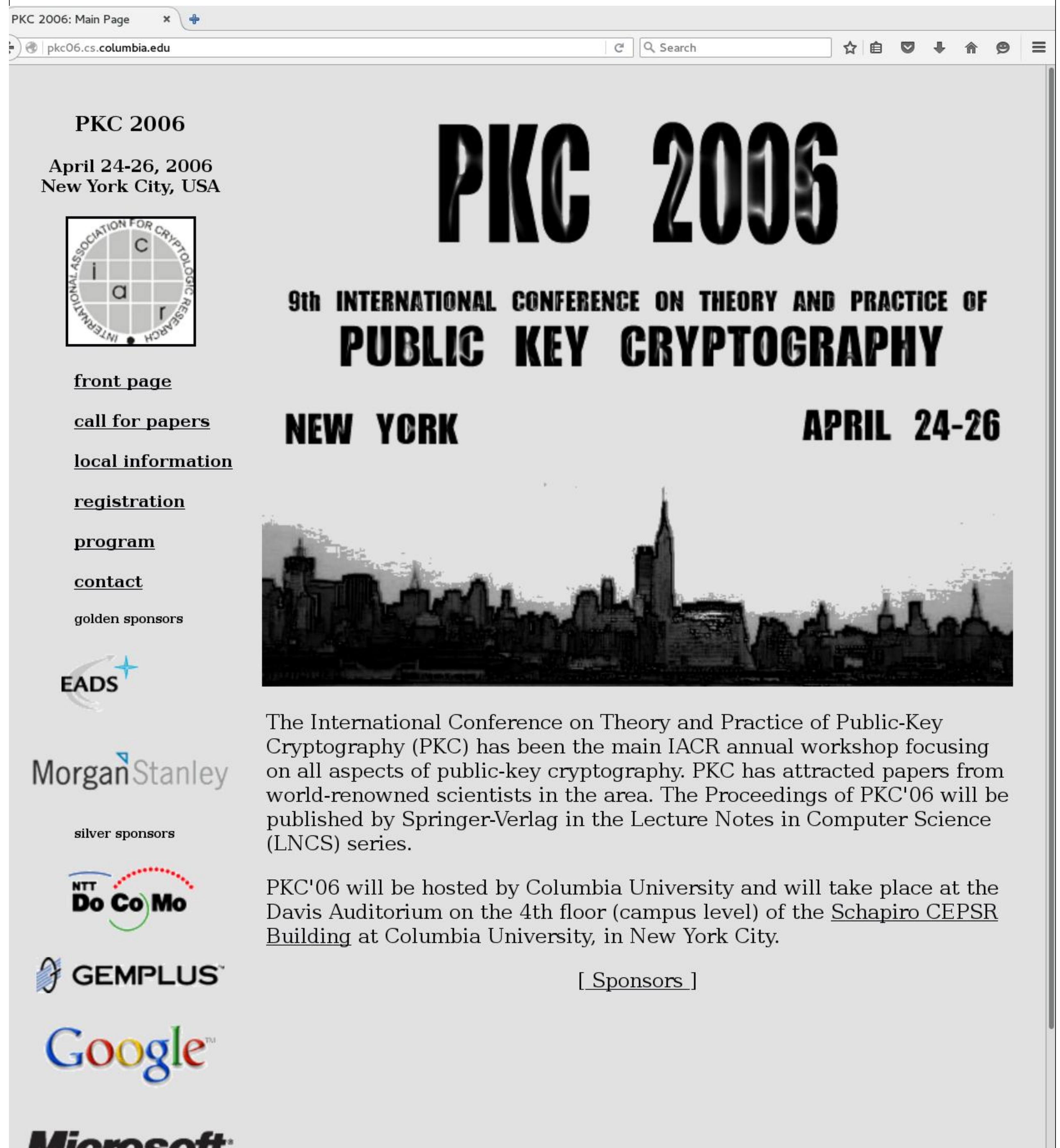


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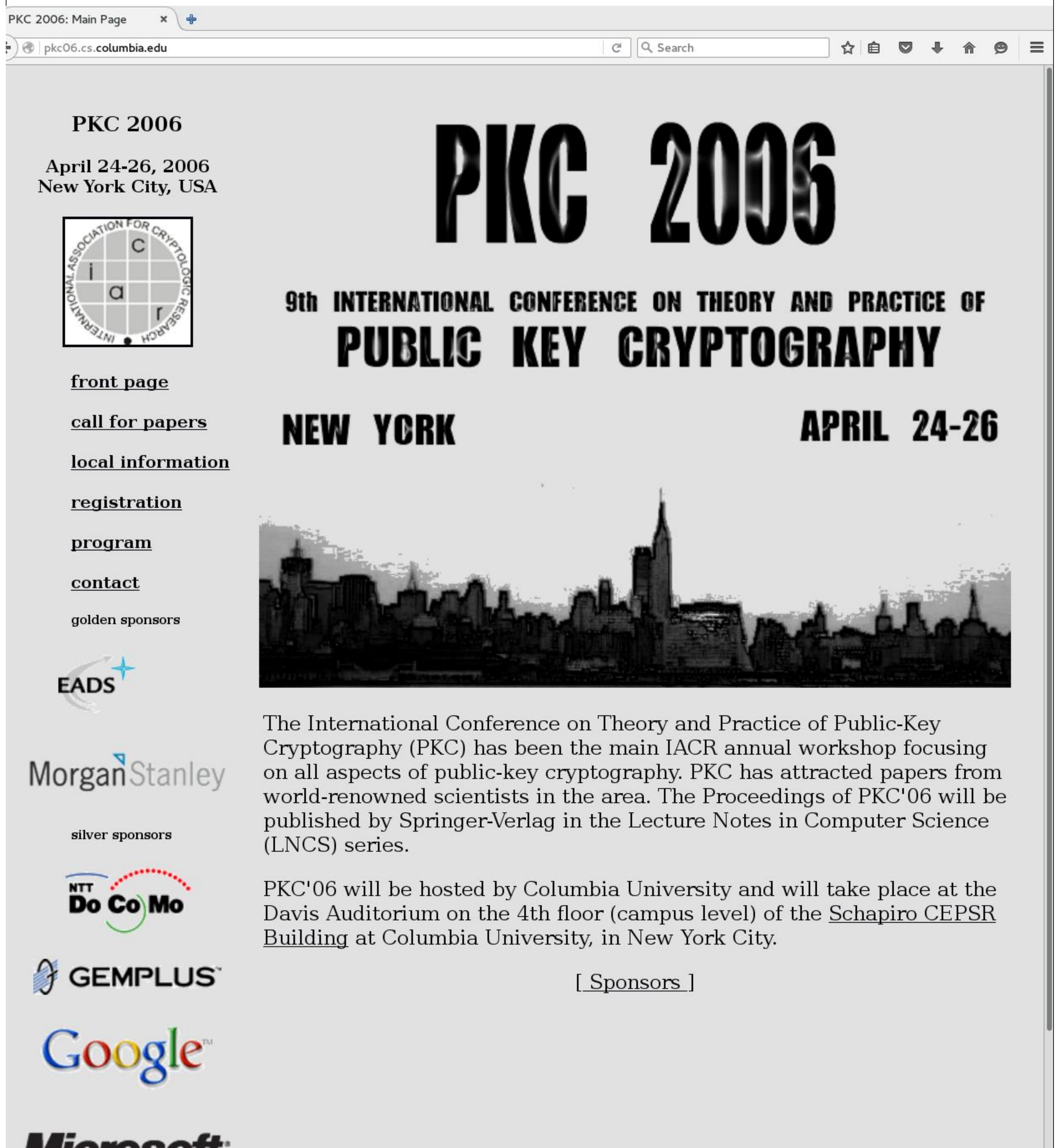


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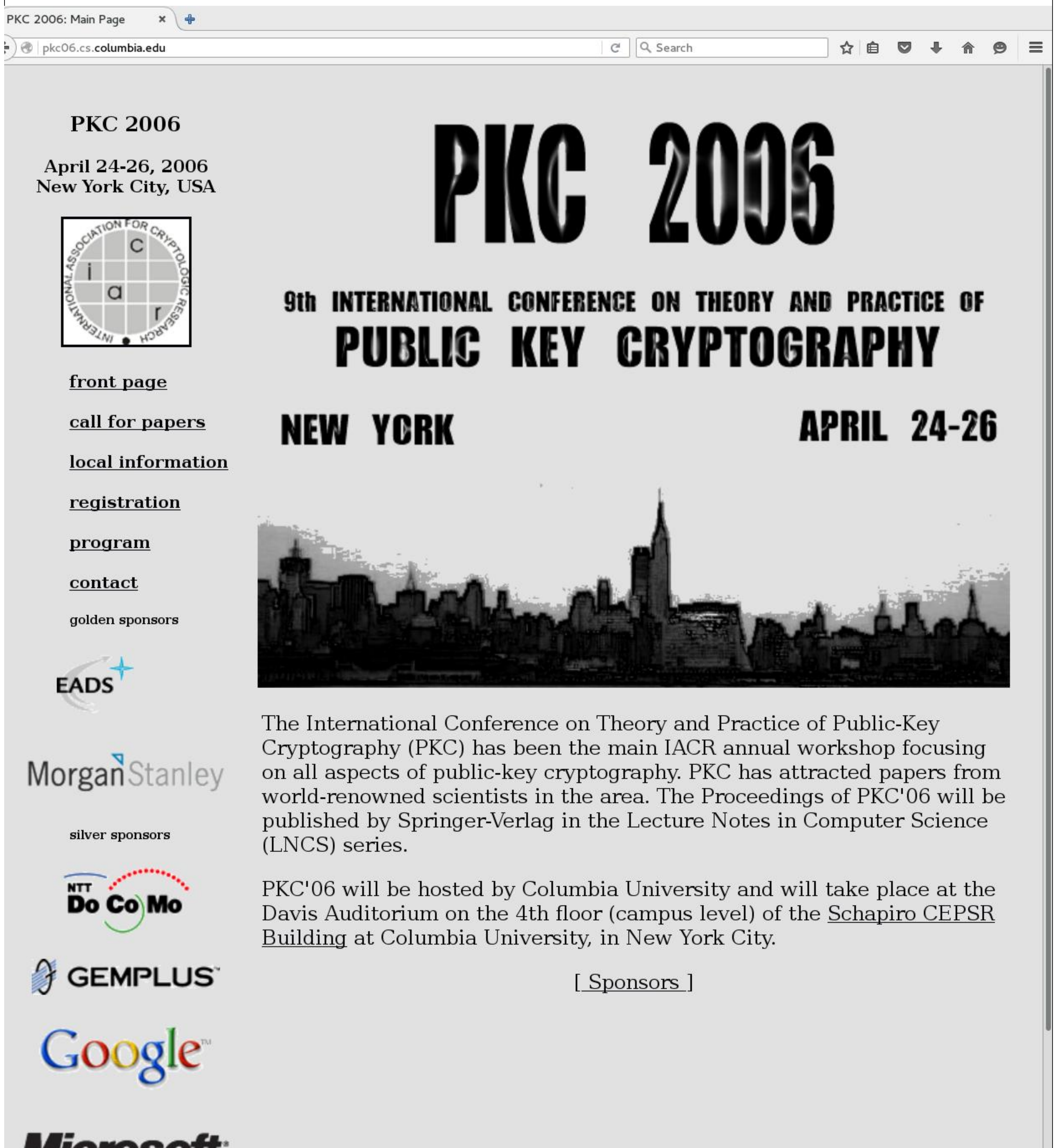


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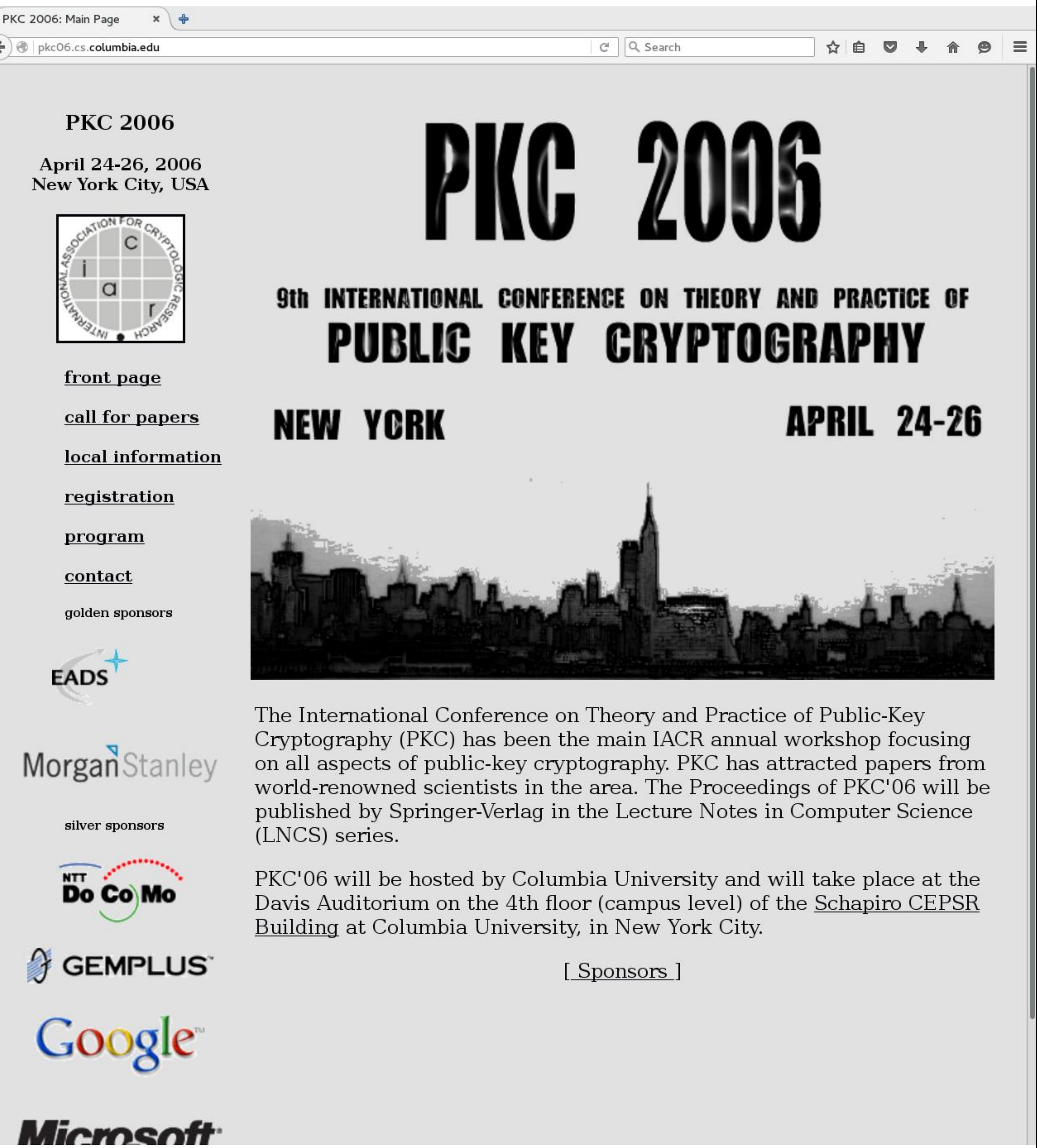




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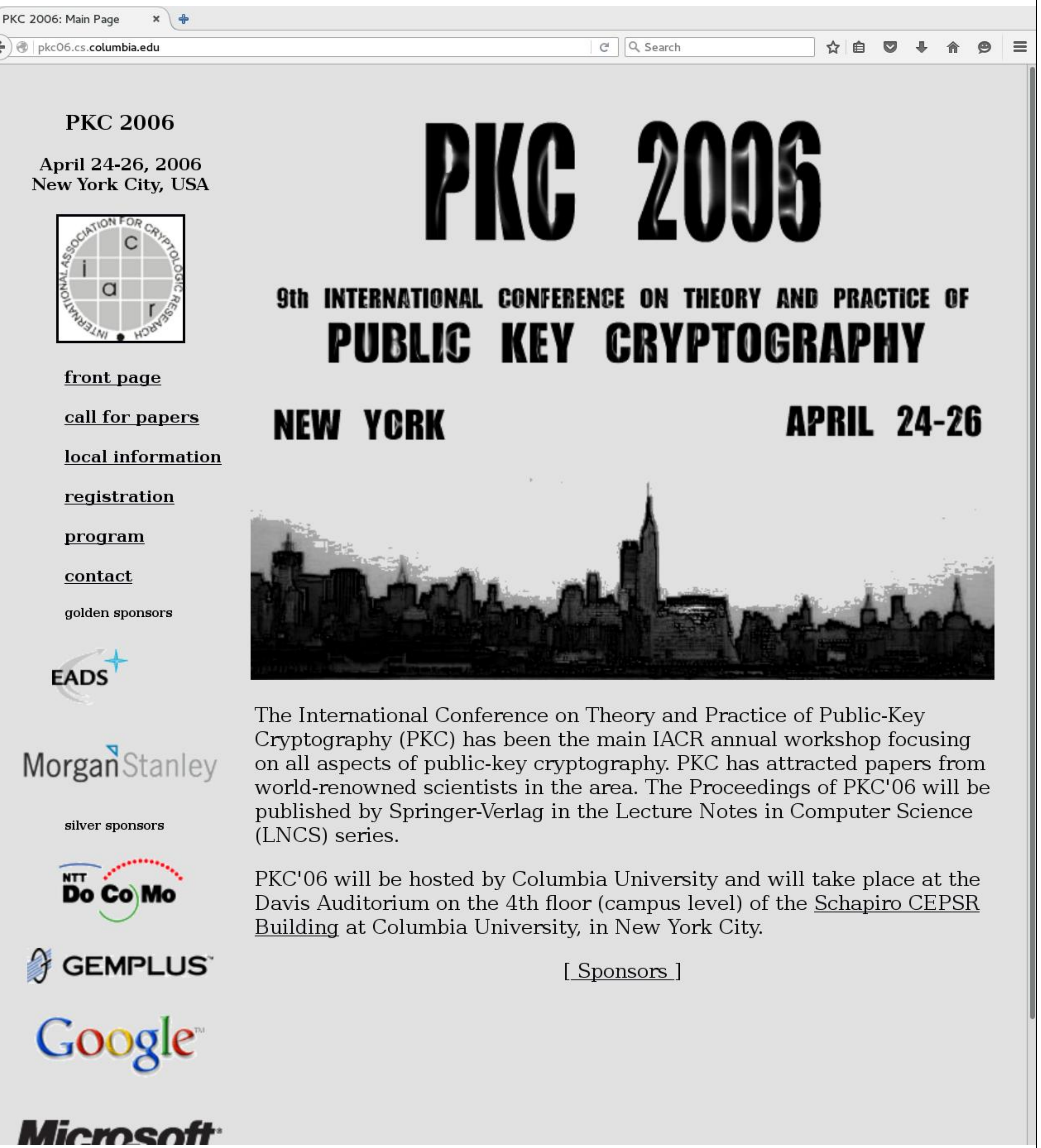




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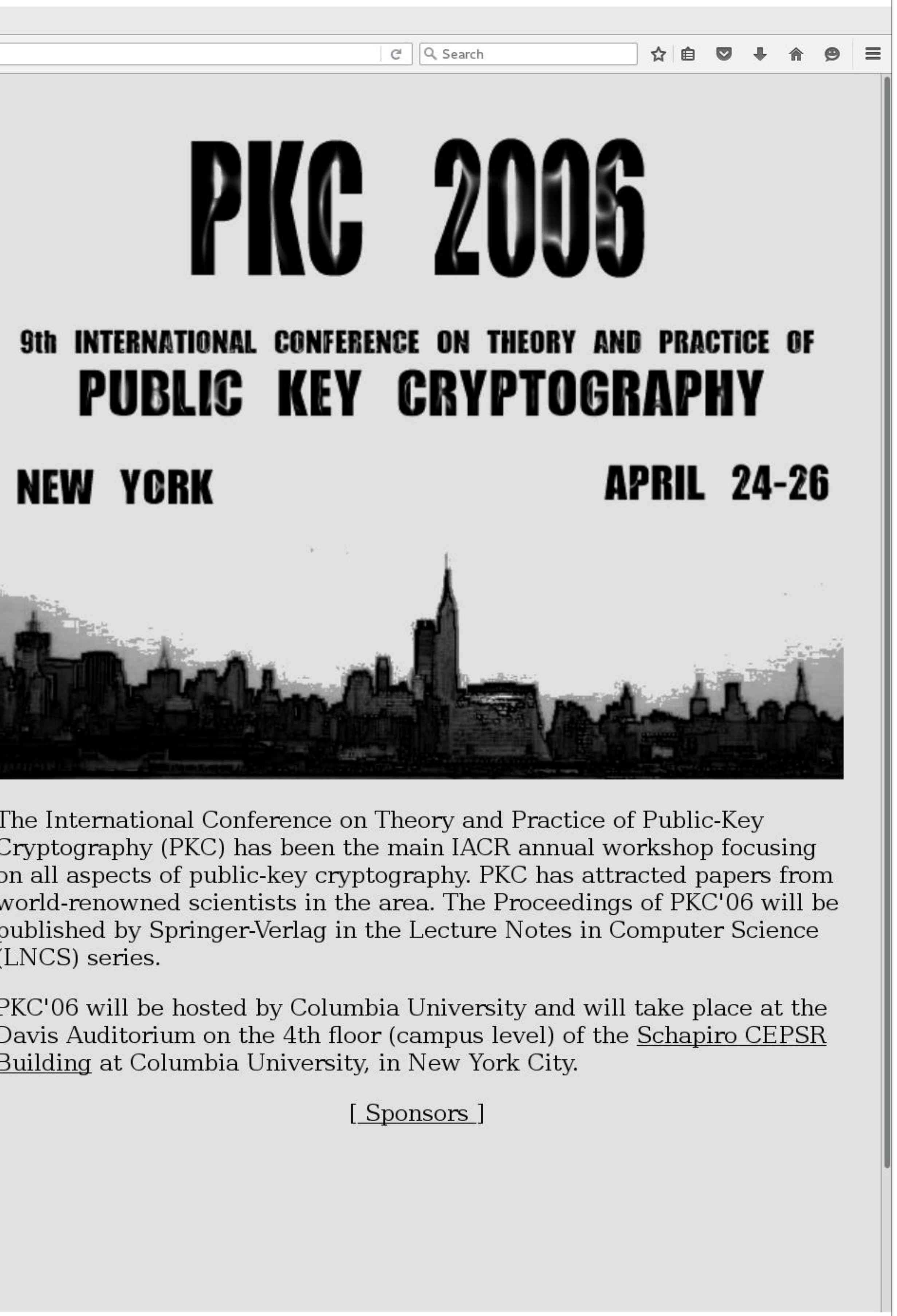




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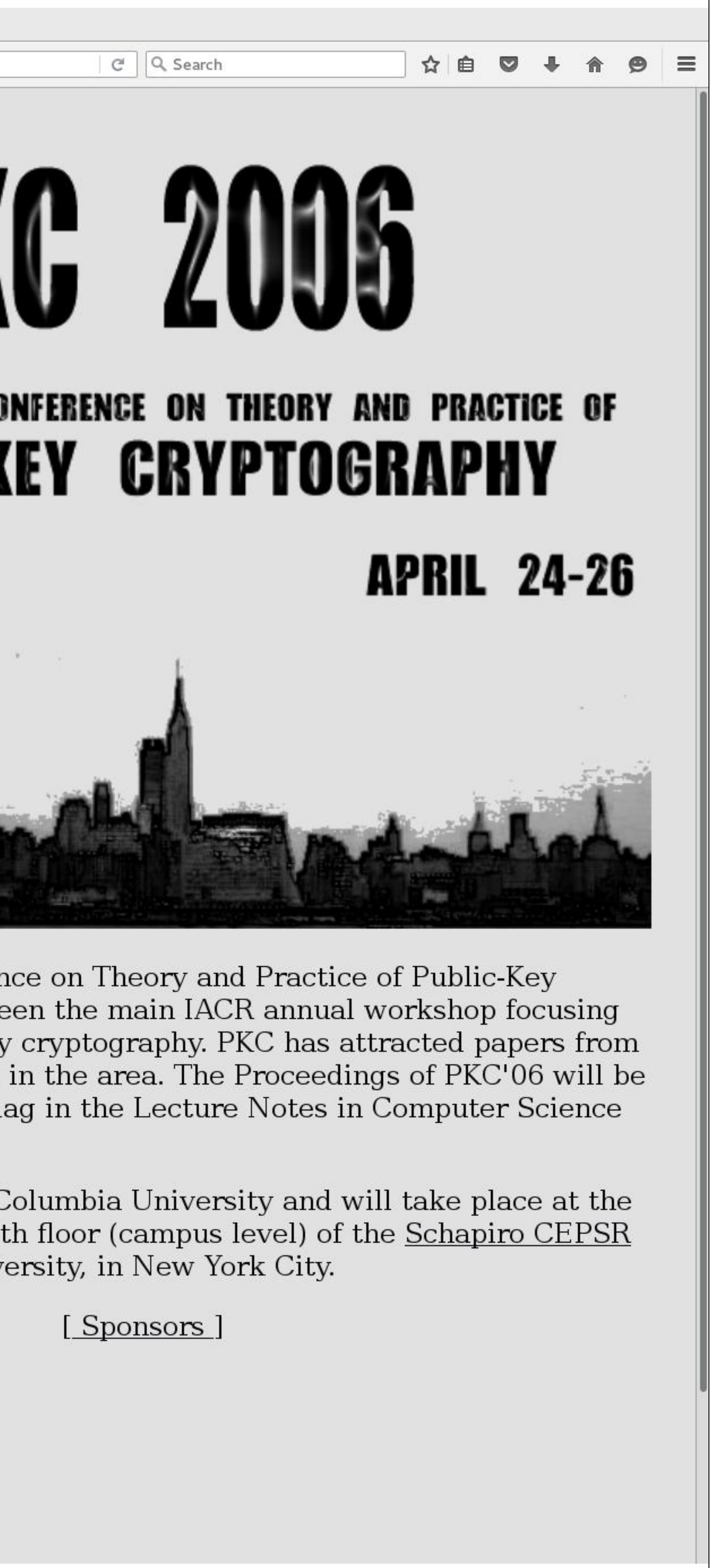




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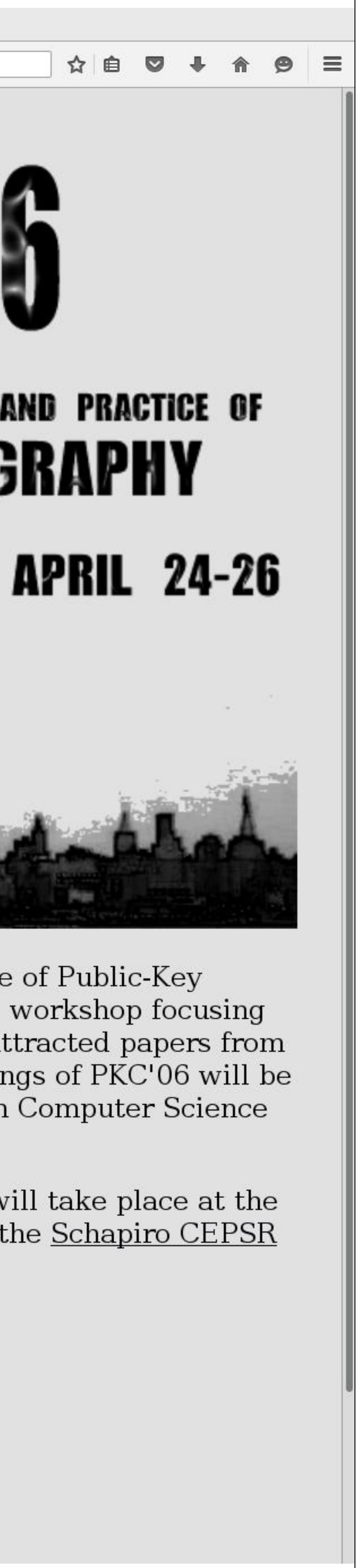




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So engineering isn't research

Email from program chairs:

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It is my pleasure to inform you that your paper "Curve25519: new Diffie-Hellman speed records" was accepted to PKC'06. Congratulations!

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Below please find the reviewers' comments on your paper "Curve25519: new Diffie-Hellman speed records" that was submitted to PKC 2006.

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Reviewer #1:

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While I think (frankly) that this is a nice engineering work, I think that this is not a "real" research paper.
I don't question the correctness but I question the appropriateness of the paper to the conference.

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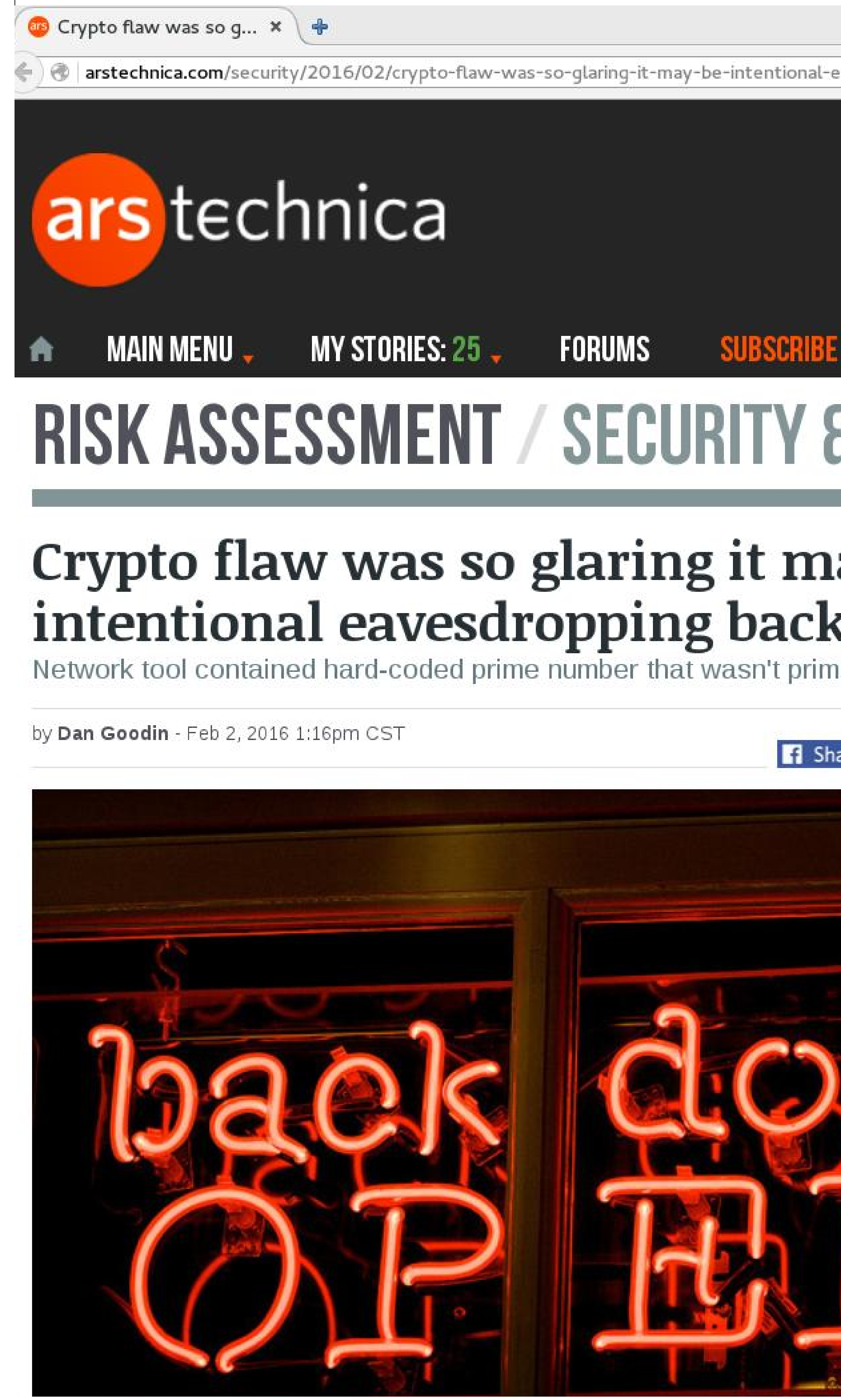
# 2016: Counterfeit “primes”.



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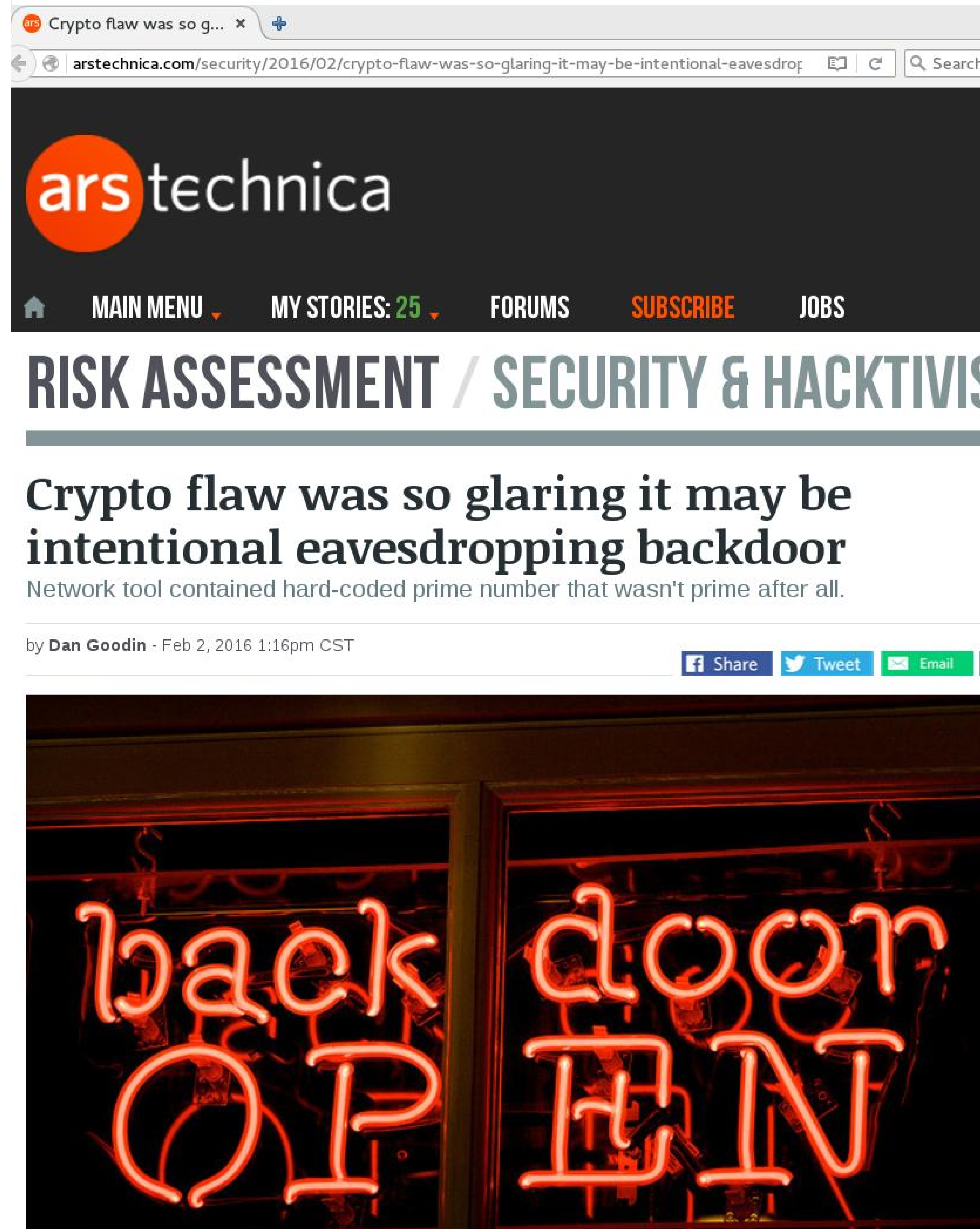

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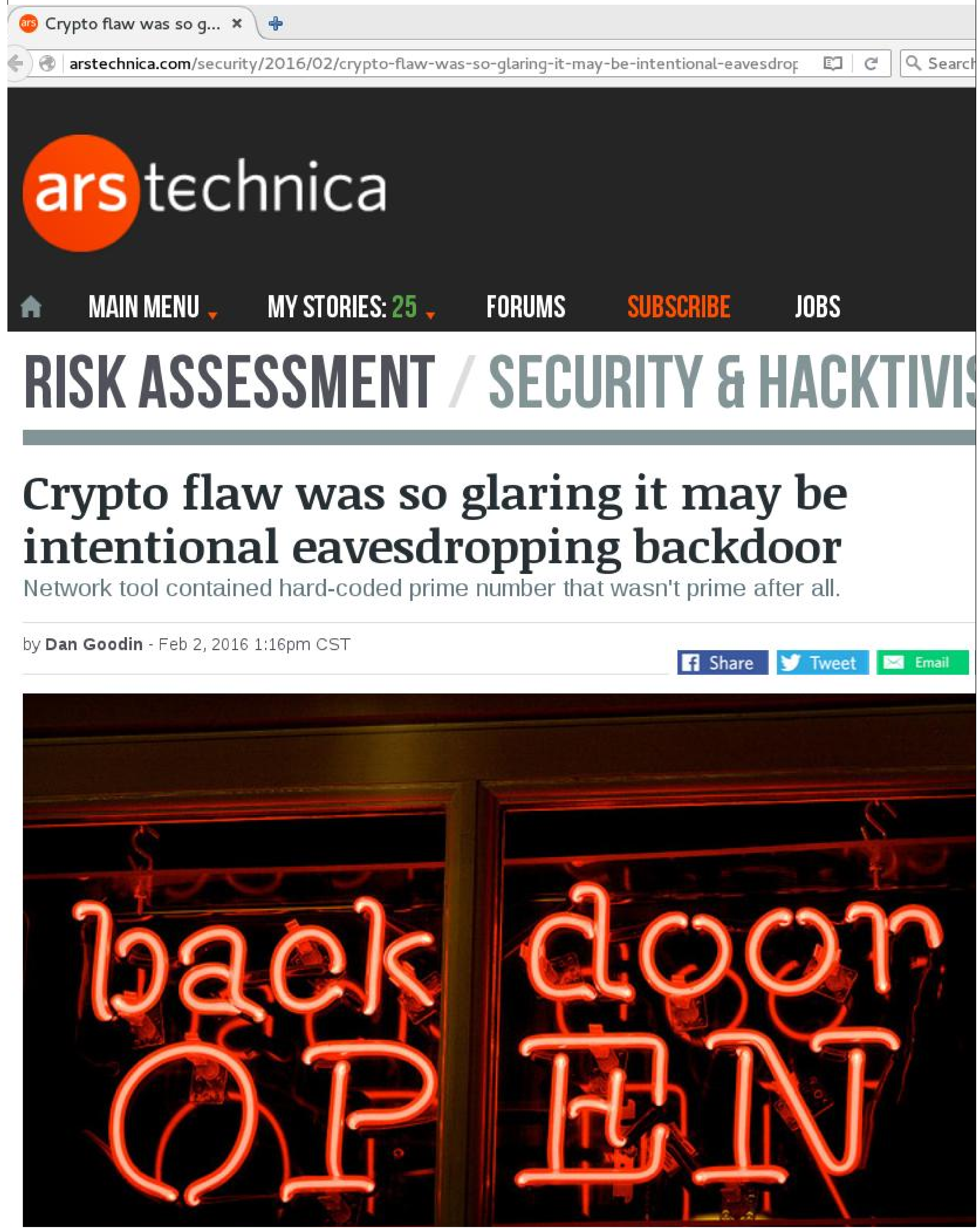


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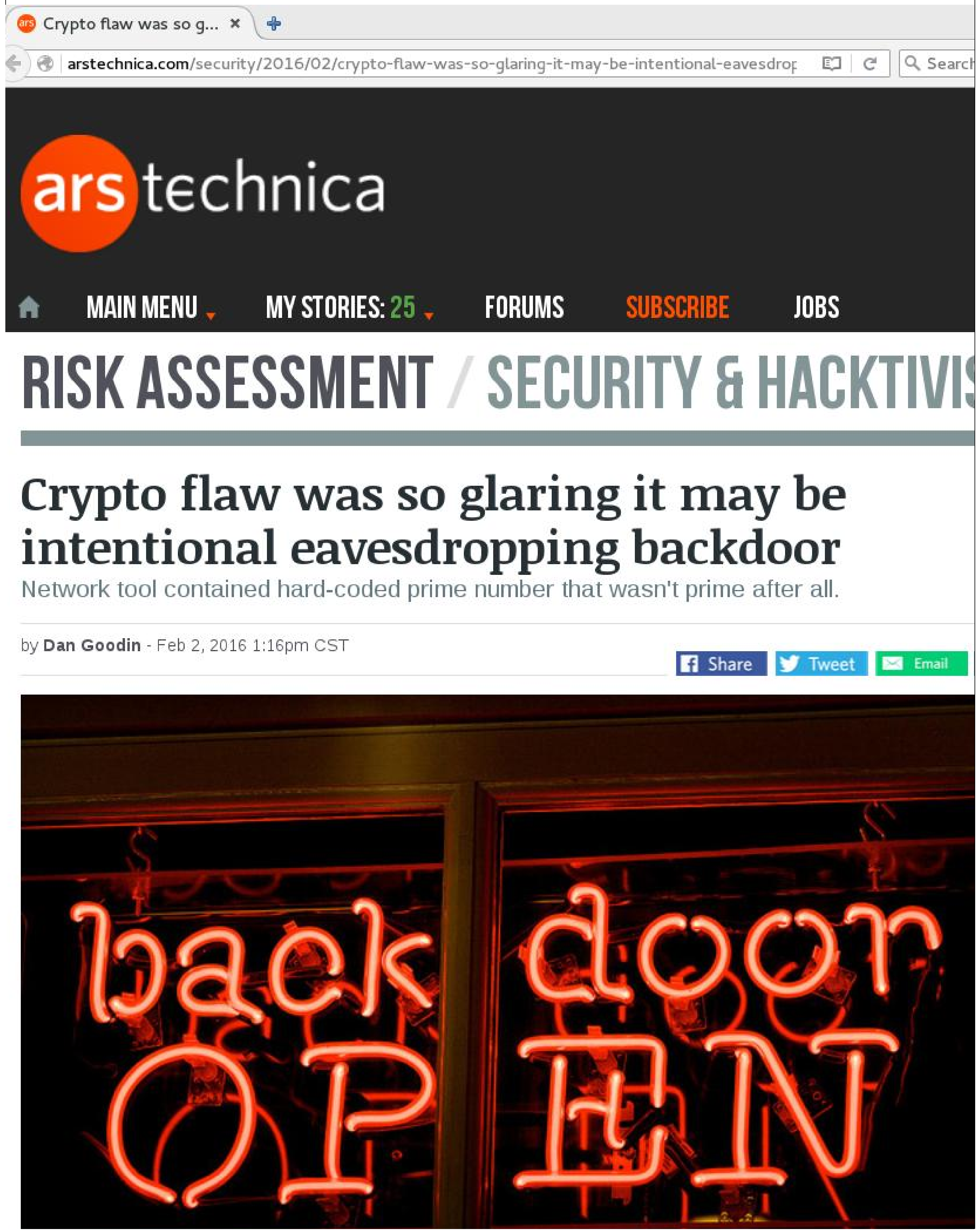

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**Crypto flaw was so glaring it may be intentional eavesdropping backdoor**

Network tool contained hard-coded prime number that wasn't prime after all.

by **Dan Goodin** - Feb 2, 2016 1:16pm CST

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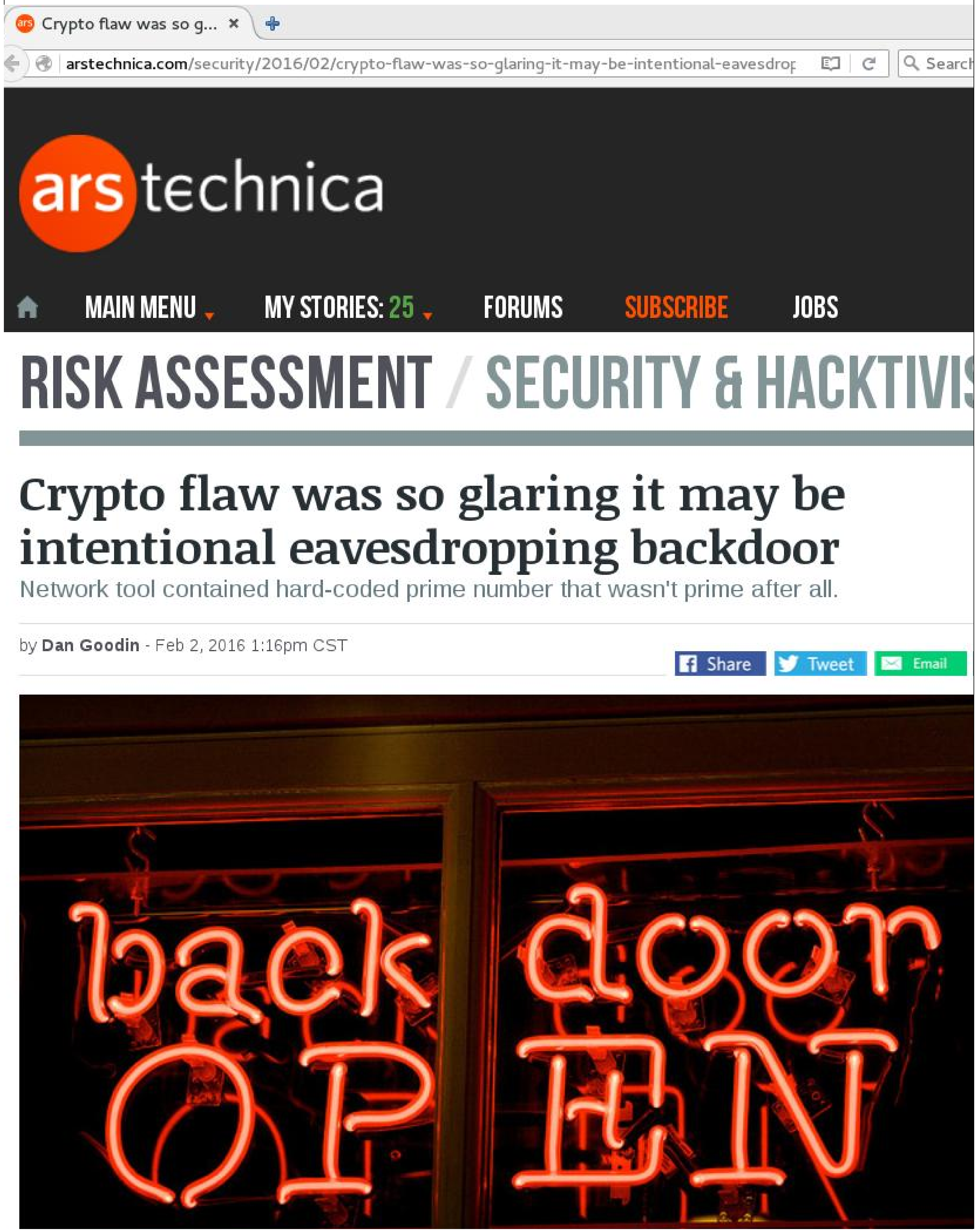
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With reviews like these,

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2016: Counterfeit “primes”.




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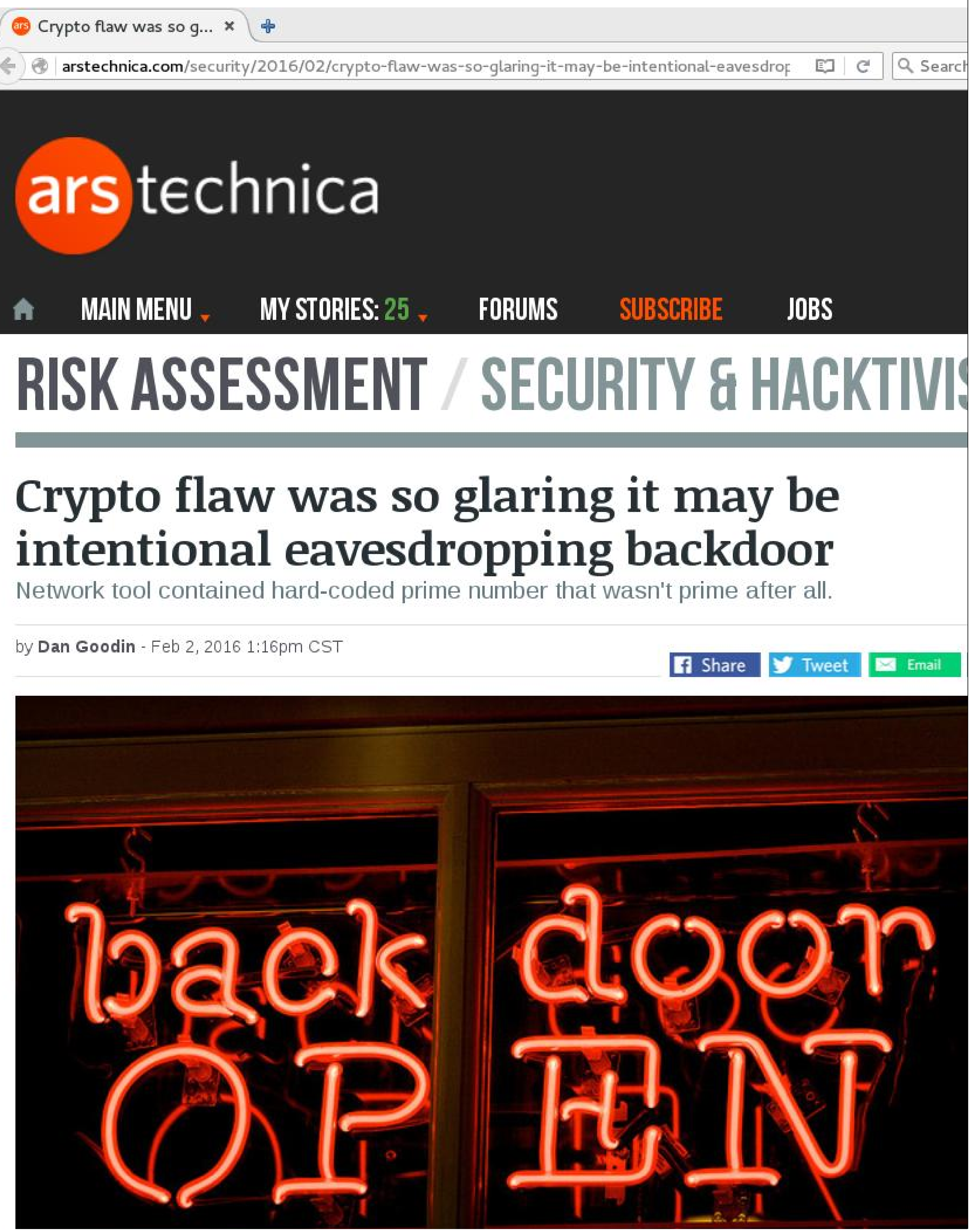




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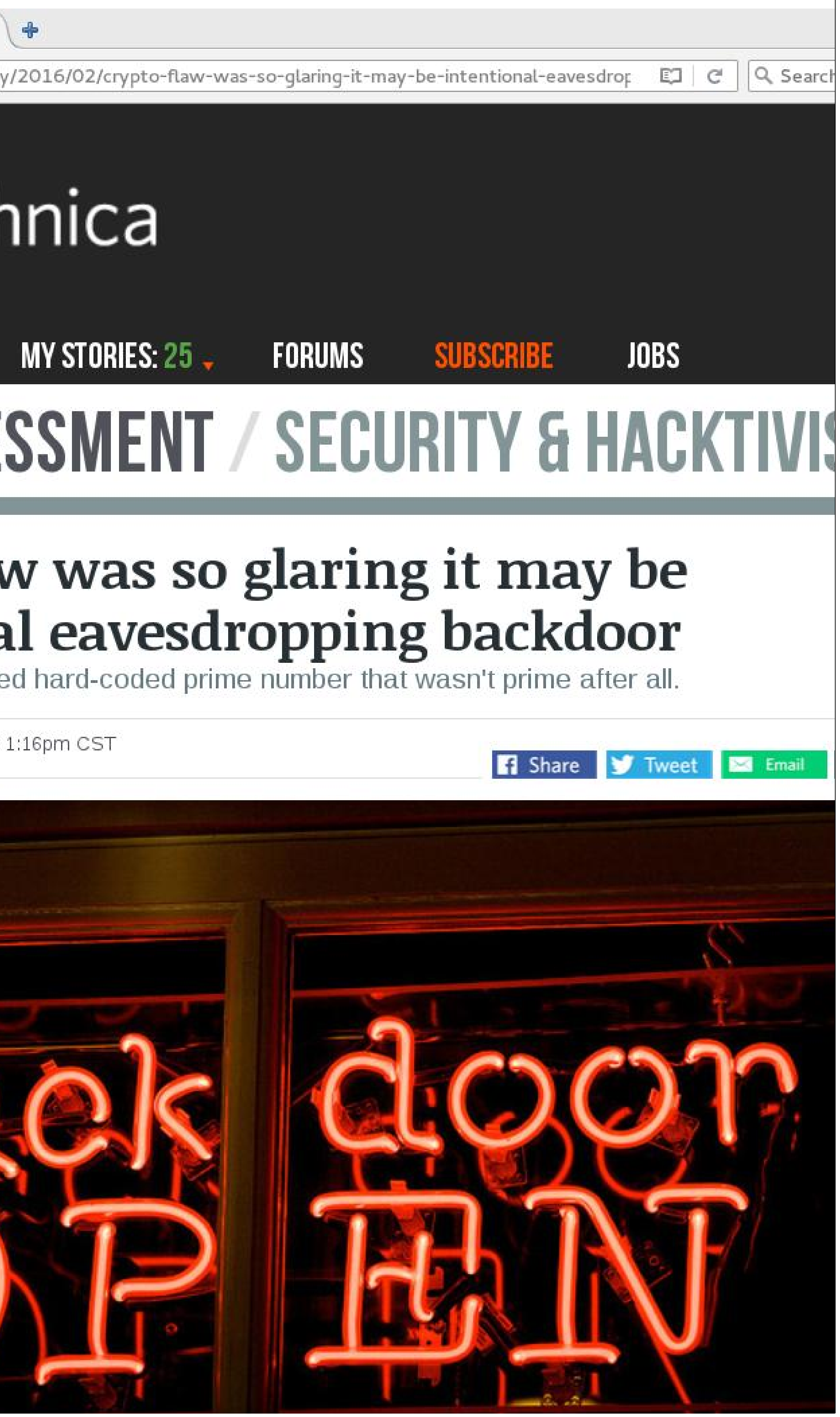




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Reviewer #4 was positive.
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Note to young cryptographers:
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New speed records for $m, n, P, Q \mapsto mP + nQ$ and other signature operations.

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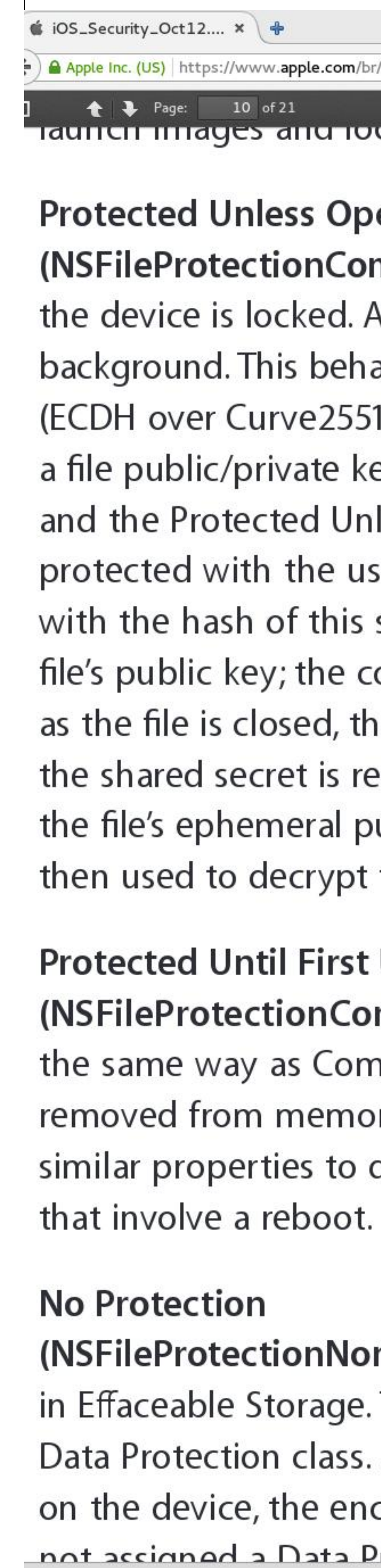
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Page: 10 of 21

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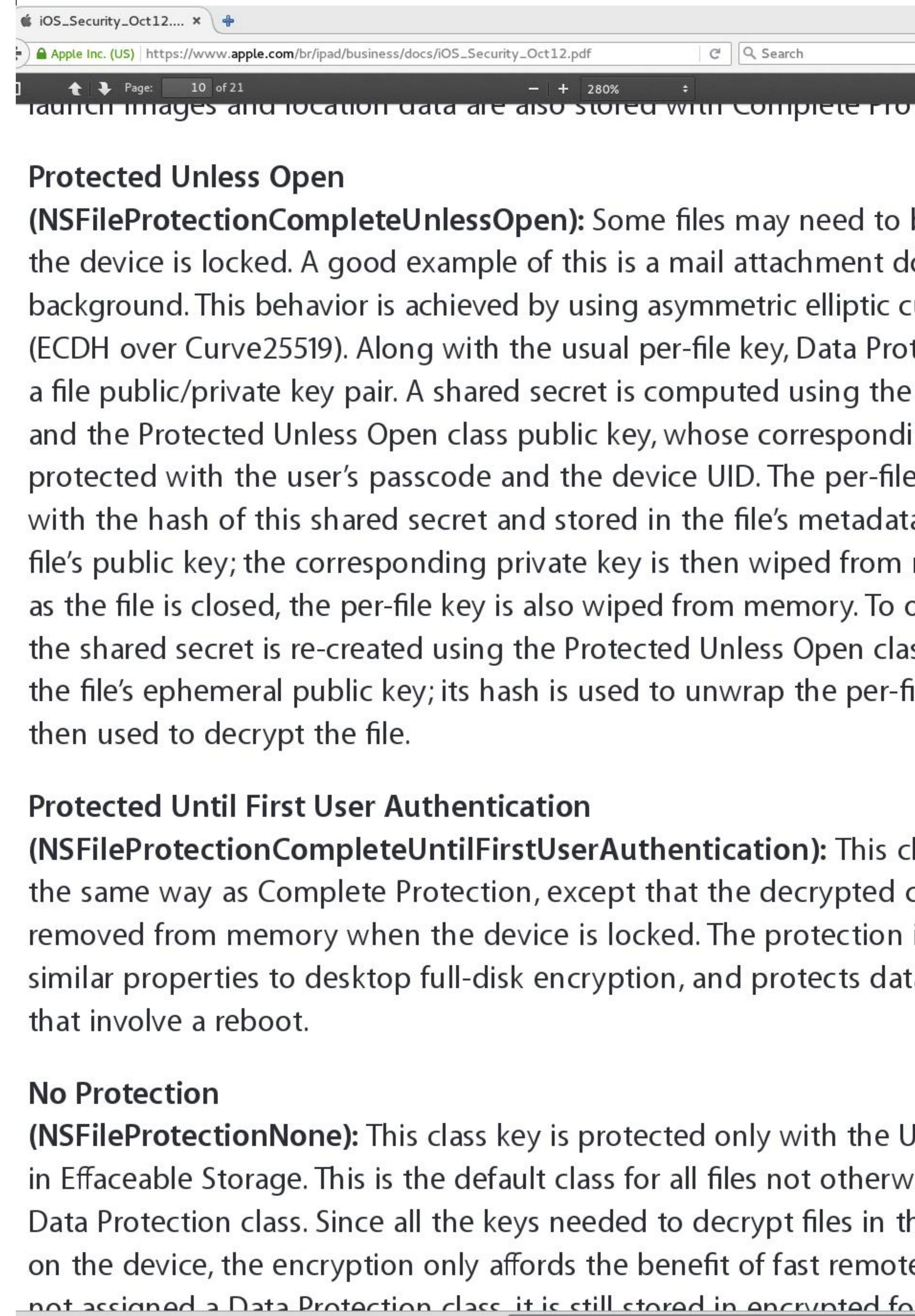
2012: Apple deploys Curve2

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## 2012: Apple deploys Curve25519

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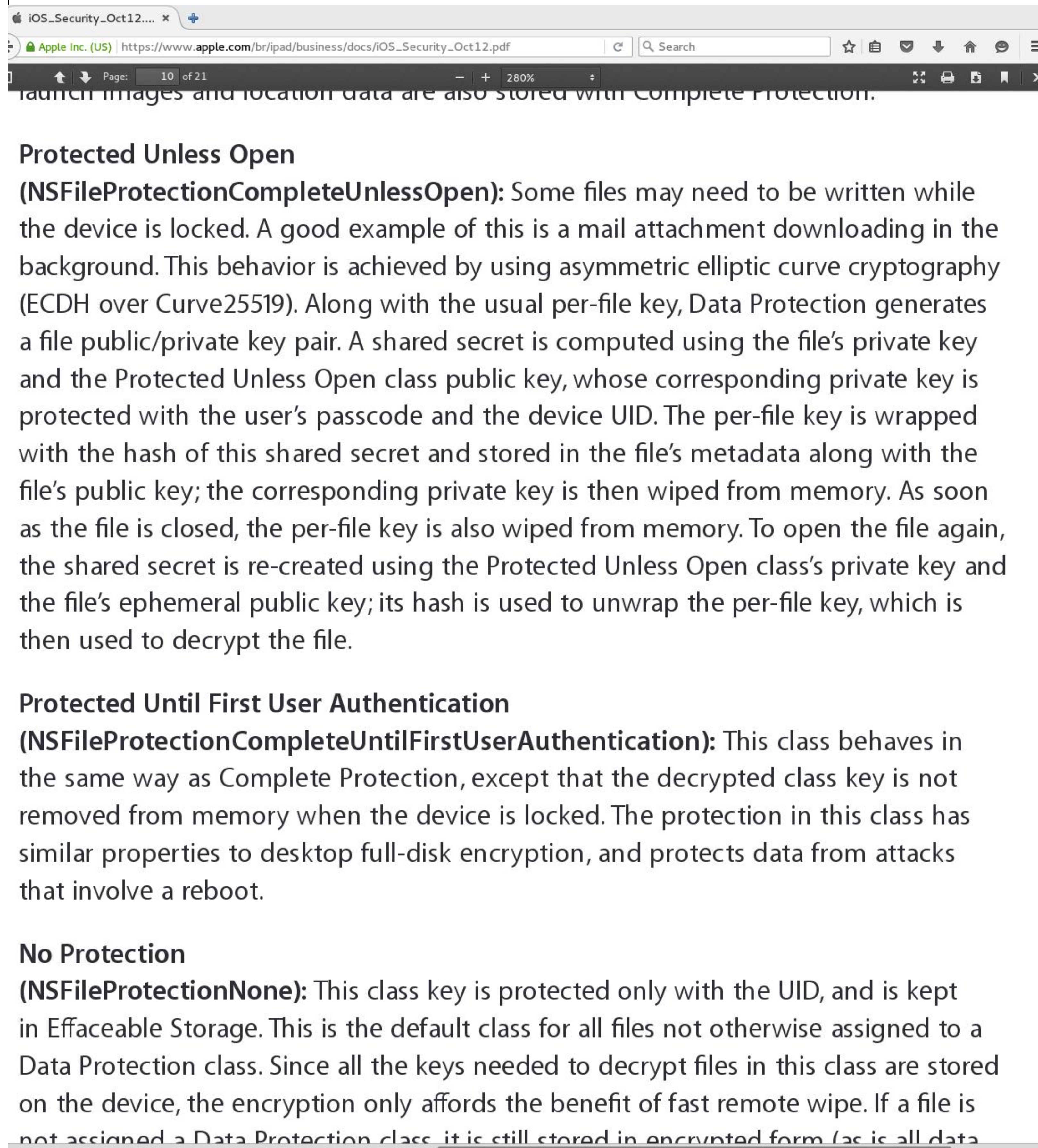


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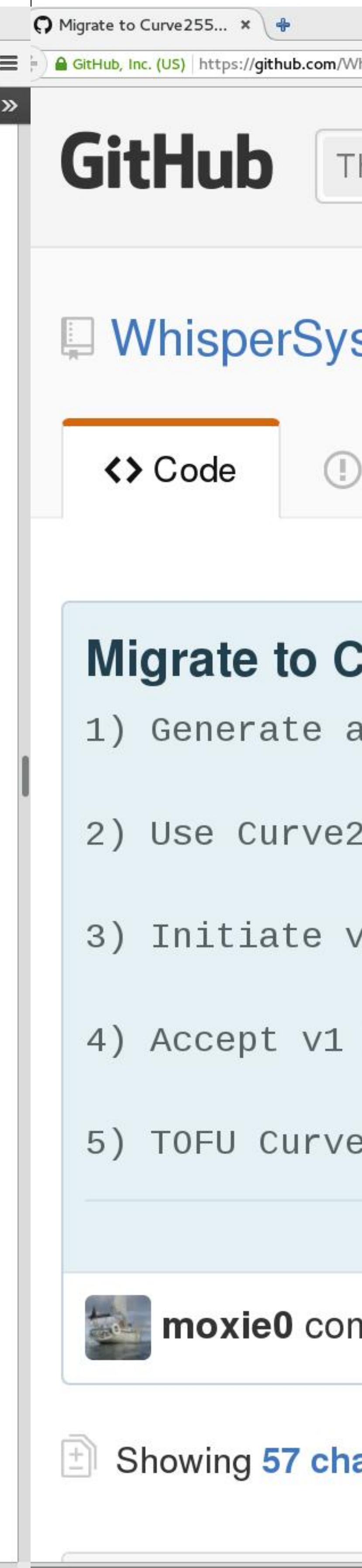

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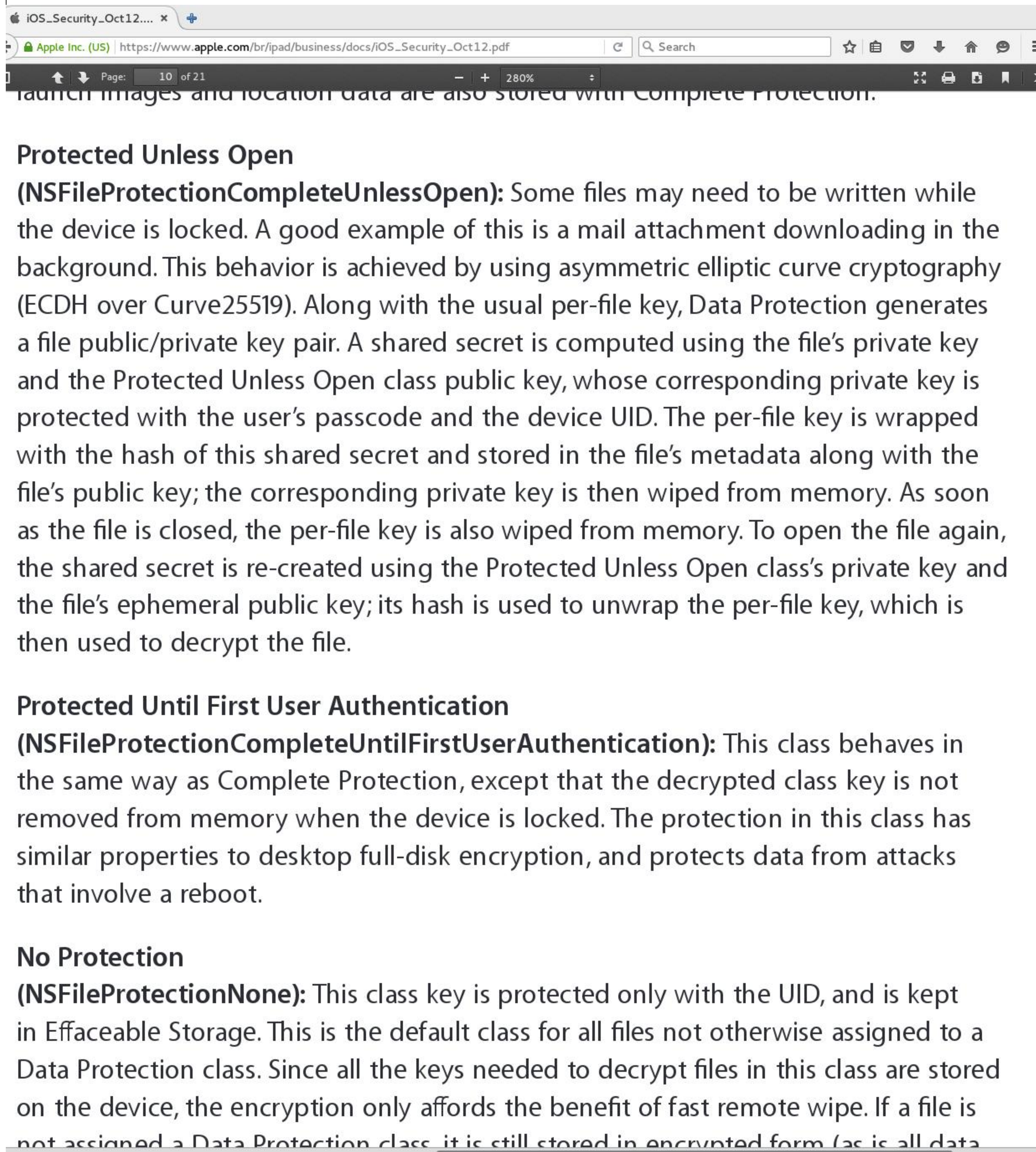


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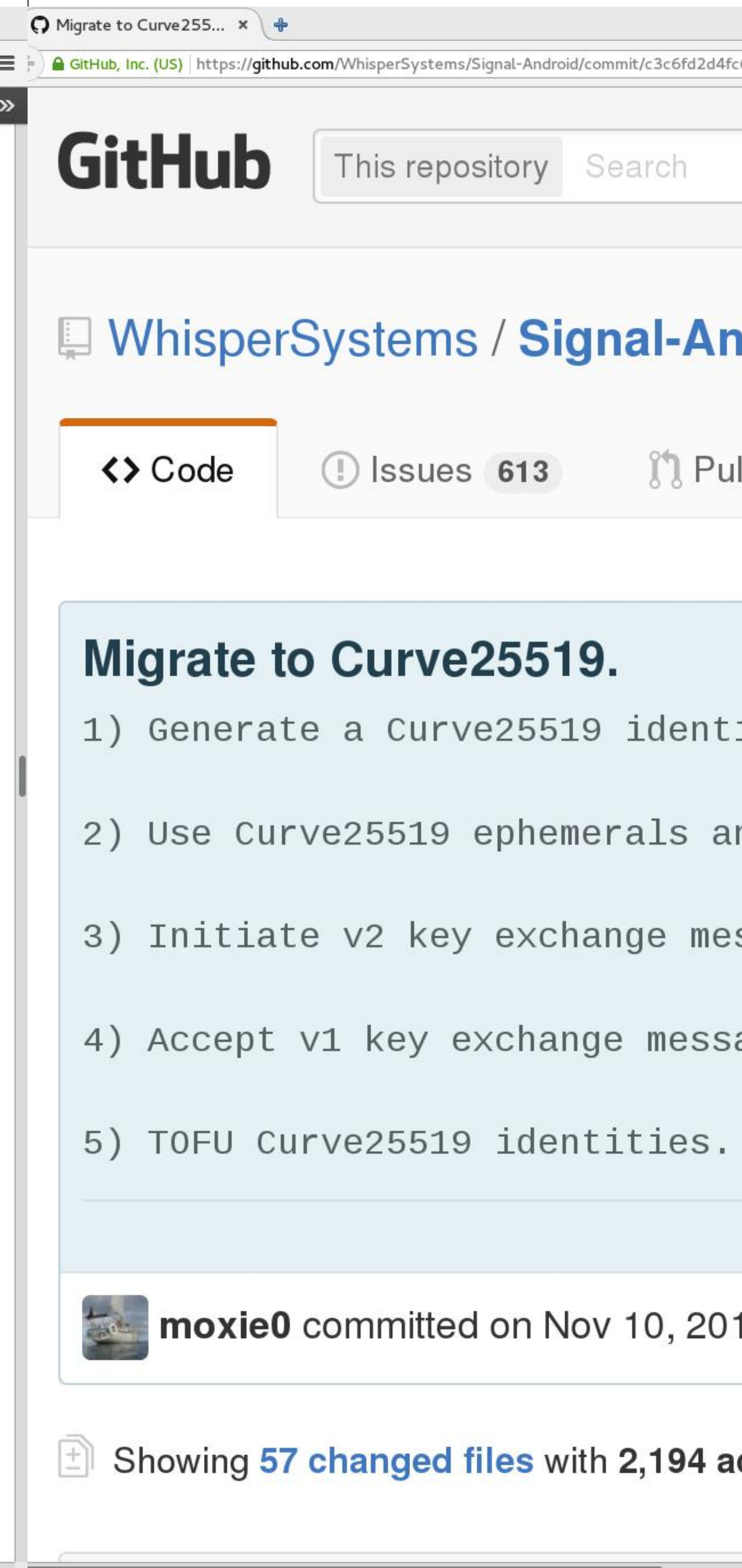

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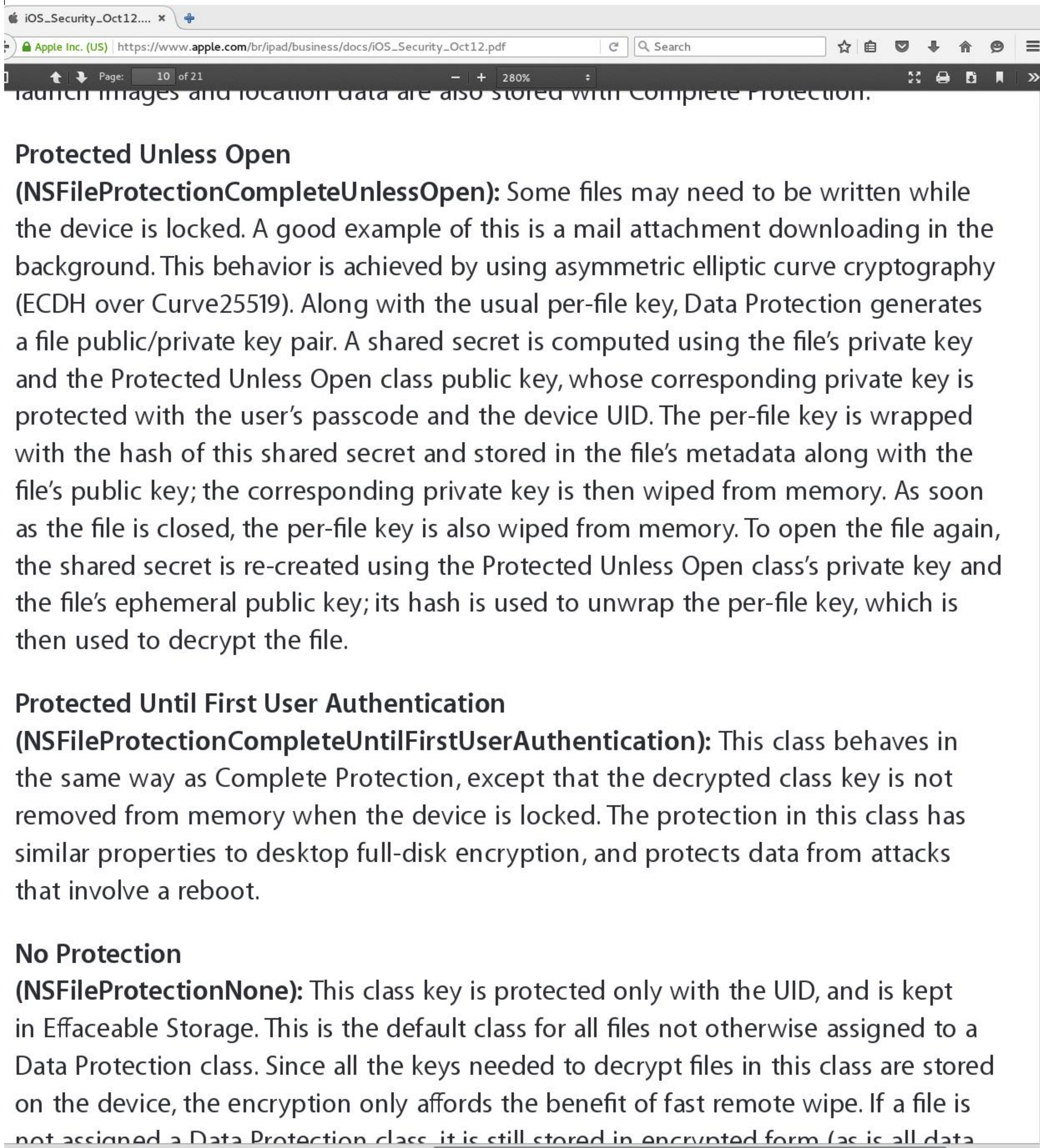
# 2012: Apple deploys Curve25519

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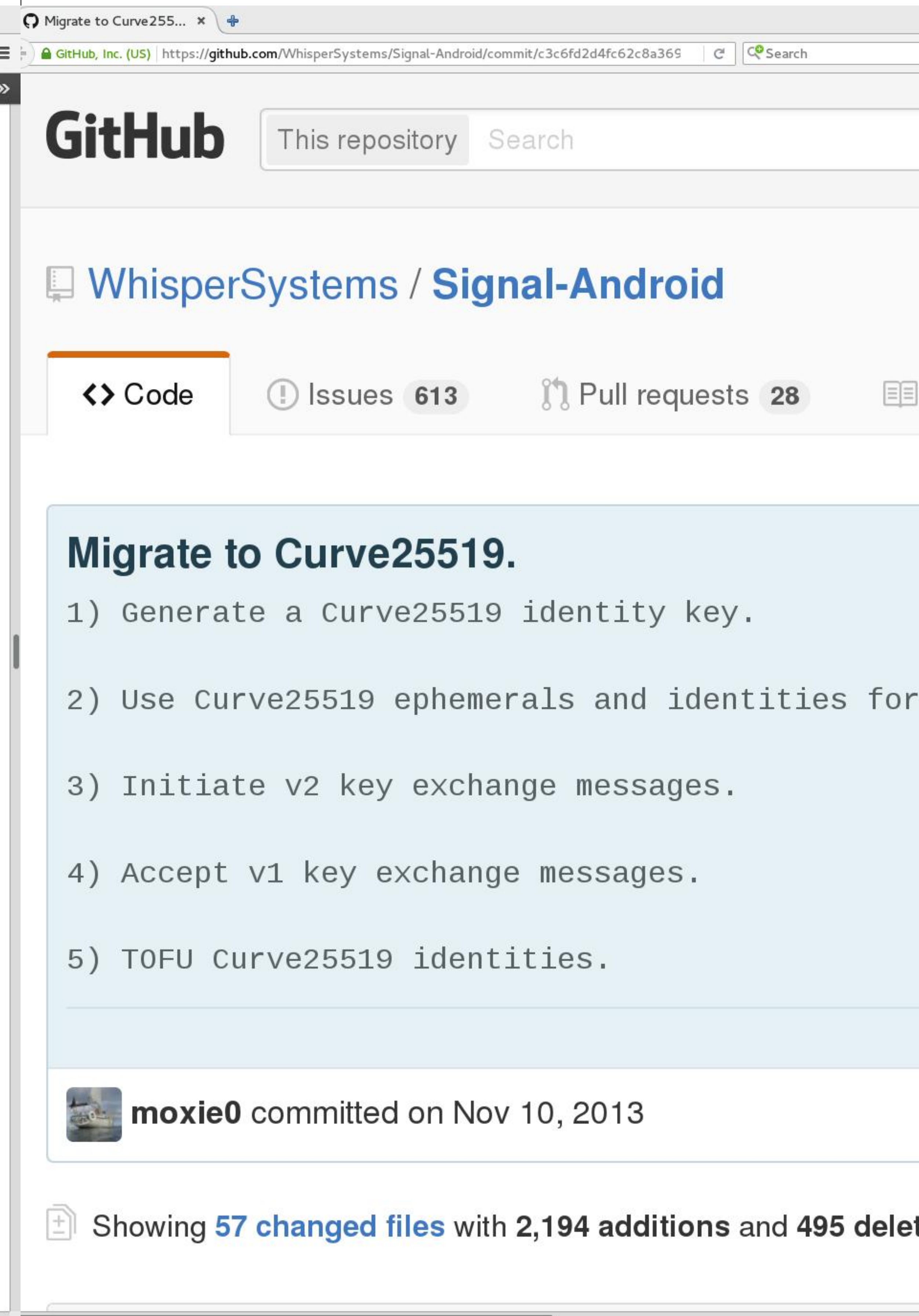
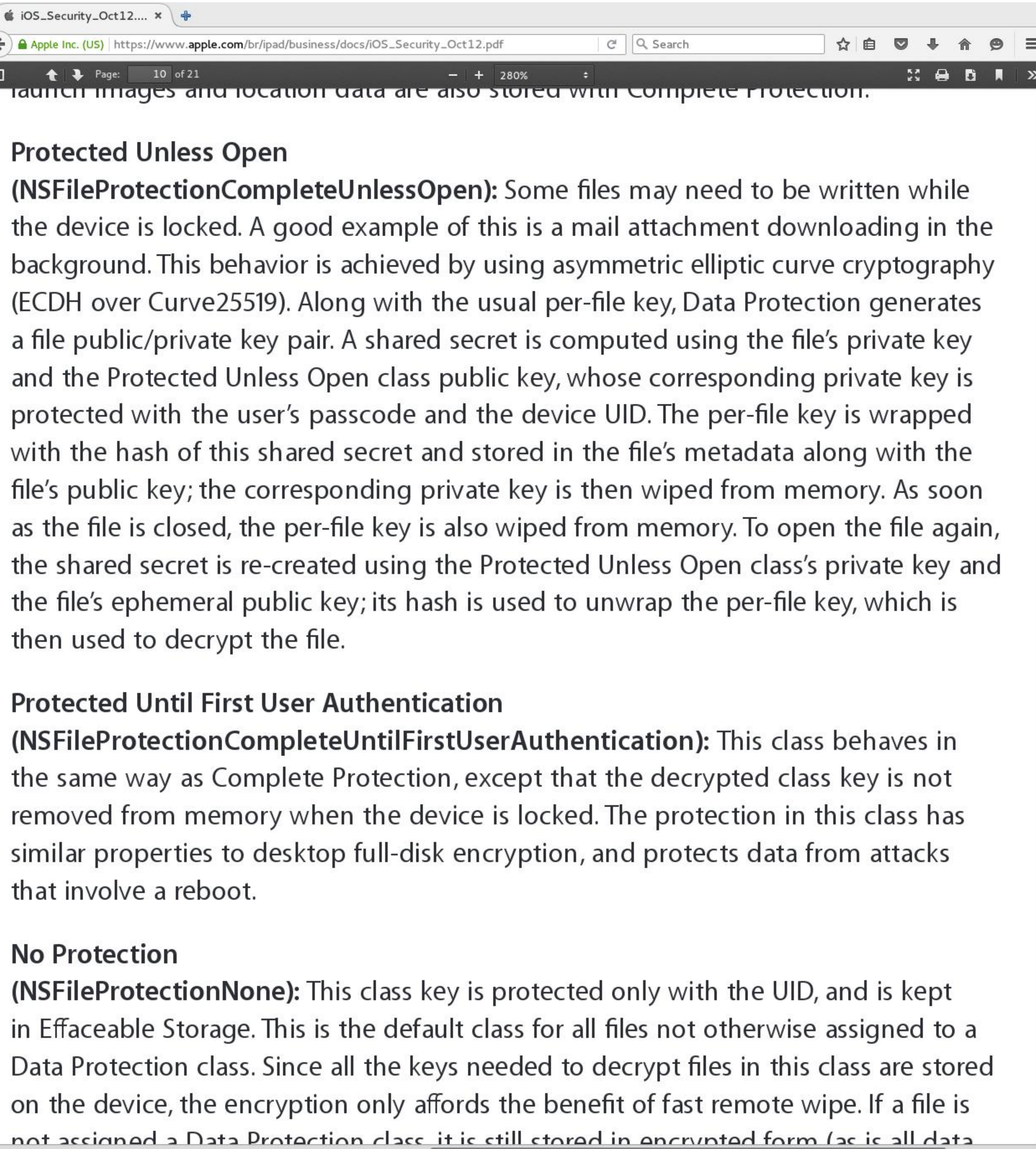
# 2013: Signal deploys Curve2



# 2012: Apple deploys Curve25519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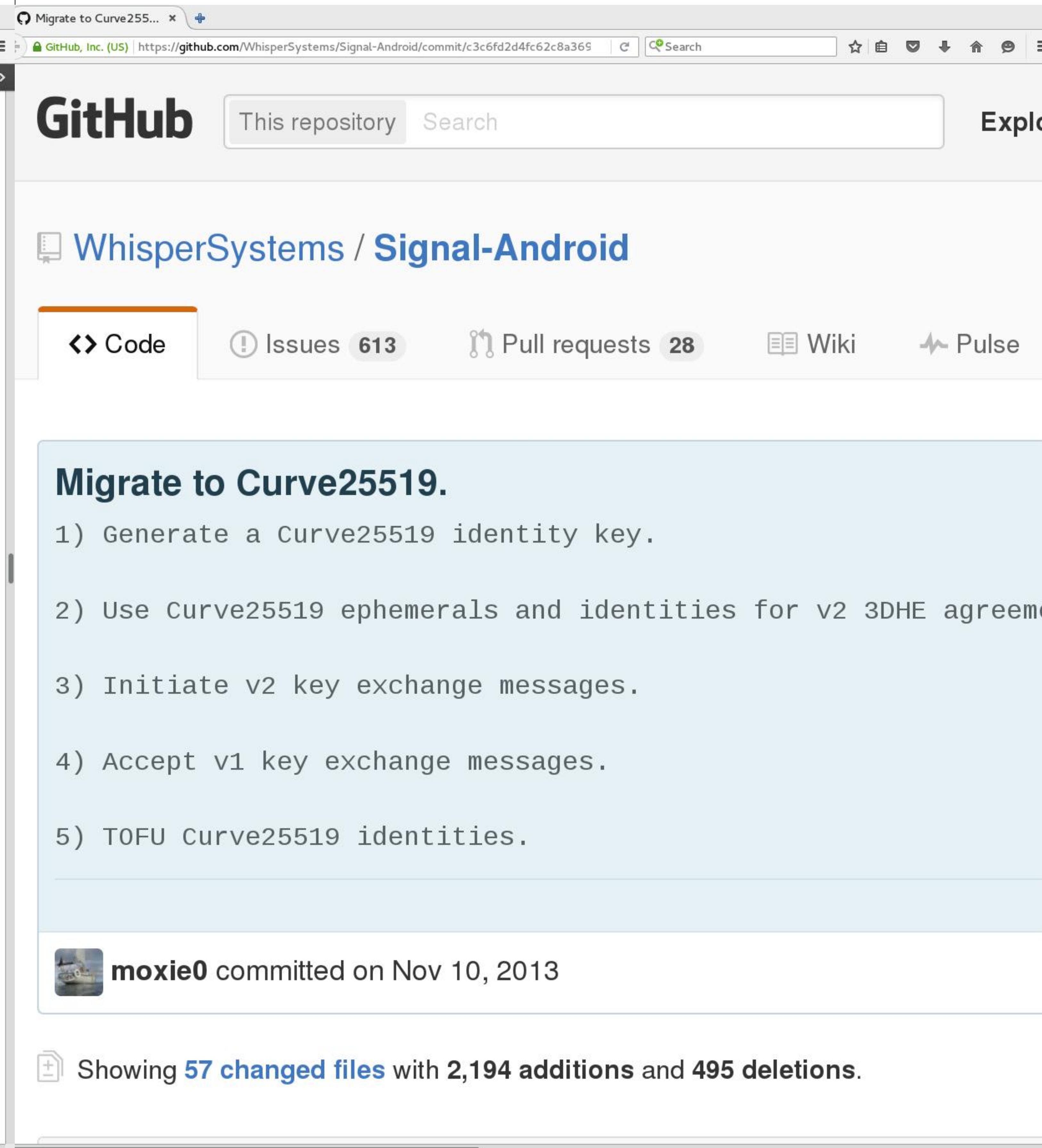

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# 2013: Signal deploys Curve25519

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WhisperSystems / **Signal-Android**

Code Issues 613 Pull requests 28 Wiki Pulse

## Migrate to Curve25519.

```
1) Generate a Curve25519 identity key.

2) Use Curve25519 ephemerals and identities for v2 3DHE agreeme

3) Initiate v2 key exchange messages.

4) Accept v1 key exchange messages.

5) TOFU Curve25519 identities.
```

**moxie0** committed on Nov 10, 2013

Showing **57 changed files** with **2,194 additions** and **495 deletions**.

# 2014: O

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Changes since OpenS
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This is a feature-f

New features:

 * ssh(1), sshd(8):
   Diffie Hellman i
   method is the de

 * ssh(1), sshd(8):
   Ed25519 is a ell
   better security
   used for both us

 * Add a new privat
   protect keys at
   Ed25519 keys, bu
   existing keys of
   We intend to mak
   Details of the n

 * ssh(1), sshd(8):
   "chacha20-poly13
   ChaCha20 stream
   encryption mode.

 * ssh(1), sshd(8):
   servers that use
   still be possibl
   DSA keys will be
   entirely in a fu

 * ssh(1), sshd(8):
   use a weaker key
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# ys Curve25519

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# 2013: Signal deploys Curve25519

Migrate to Curve255...

GitHub, Inc. (US) https://github.com/WhisperSystems/Signal-Android/commit/c3c6fd2d4fc62c8a365

GitHub This repository Search Explo

WhisperSystems / **Signal-Android**

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# 2014: OpenSSH d

http://www...lease-6.5

www.openssh.com/txt/release-6.5

```
Changes since OpenSSH 6.4
=========================

This is a feature-focused release.

New features:

 * ssh(1), sshd(8): Add support for k
   Diffie Hellman in Daniel Bernstein
   method is the default when both th

 * ssh(1), sshd(8): Add support for E
   Ed25519 is a elliptic curve signat
   better security than ECDSA and DSA
   used for both user and host keys.

 * Add a new private key format that
   protect keys at rest. This format
   Ed25519 keys, but may be requested
   existing keys of other types via t
   We intend to make the new format t
   Details of the new format are in t

 * ssh(1), sshd(8): Add a new transpo
   "chacha20-poly1305@openssh.com" th
   ChaCha20 stream cipher and Poly130
   encryption mode. Details are in th

 * ssh(1), sshd(8): Refuse RSA keys f
   servers that use the obsolete RSA+
   still be possible to connect with
   DSA keys will be accepted, and Ope
   entirely in a future release.

 * ssh(1), sshd(8): Refuse old propri
   use a weaker key exchange hash cal
```

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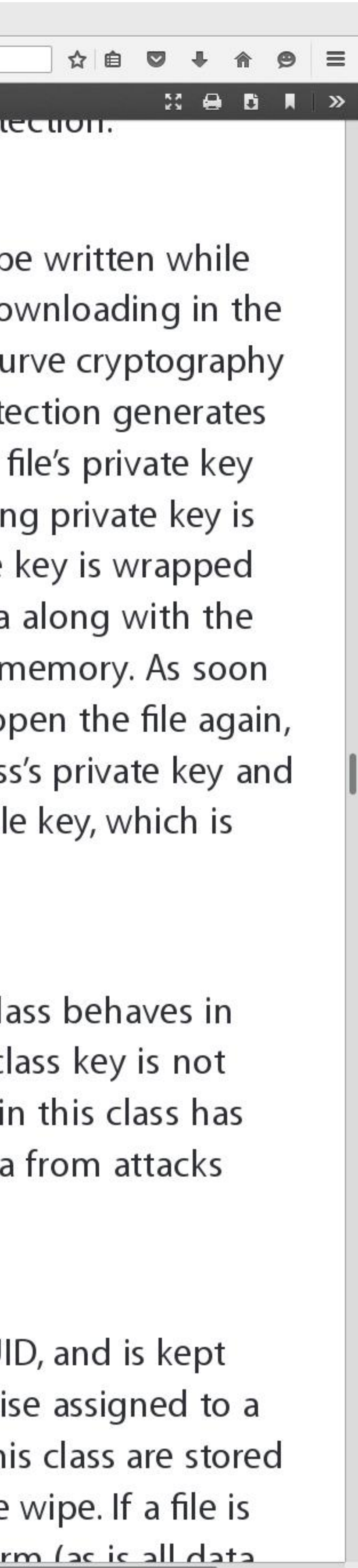
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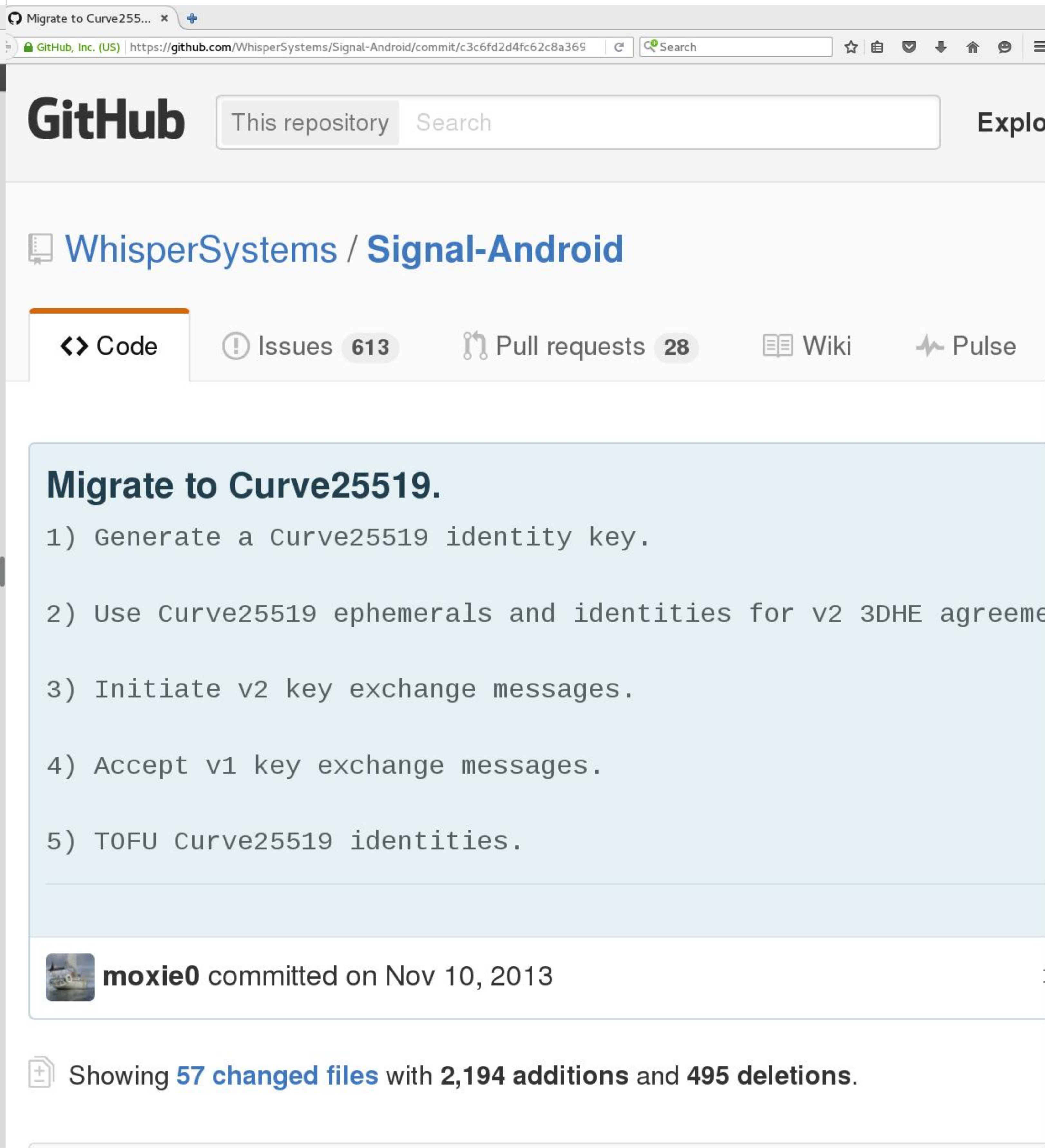
ng private key is
e key is wrapped
a along with the
memory. As soon
open the file again,
ss's private key and
le key, which is

lass behaves in
class key is not
in this class has
a from attacks

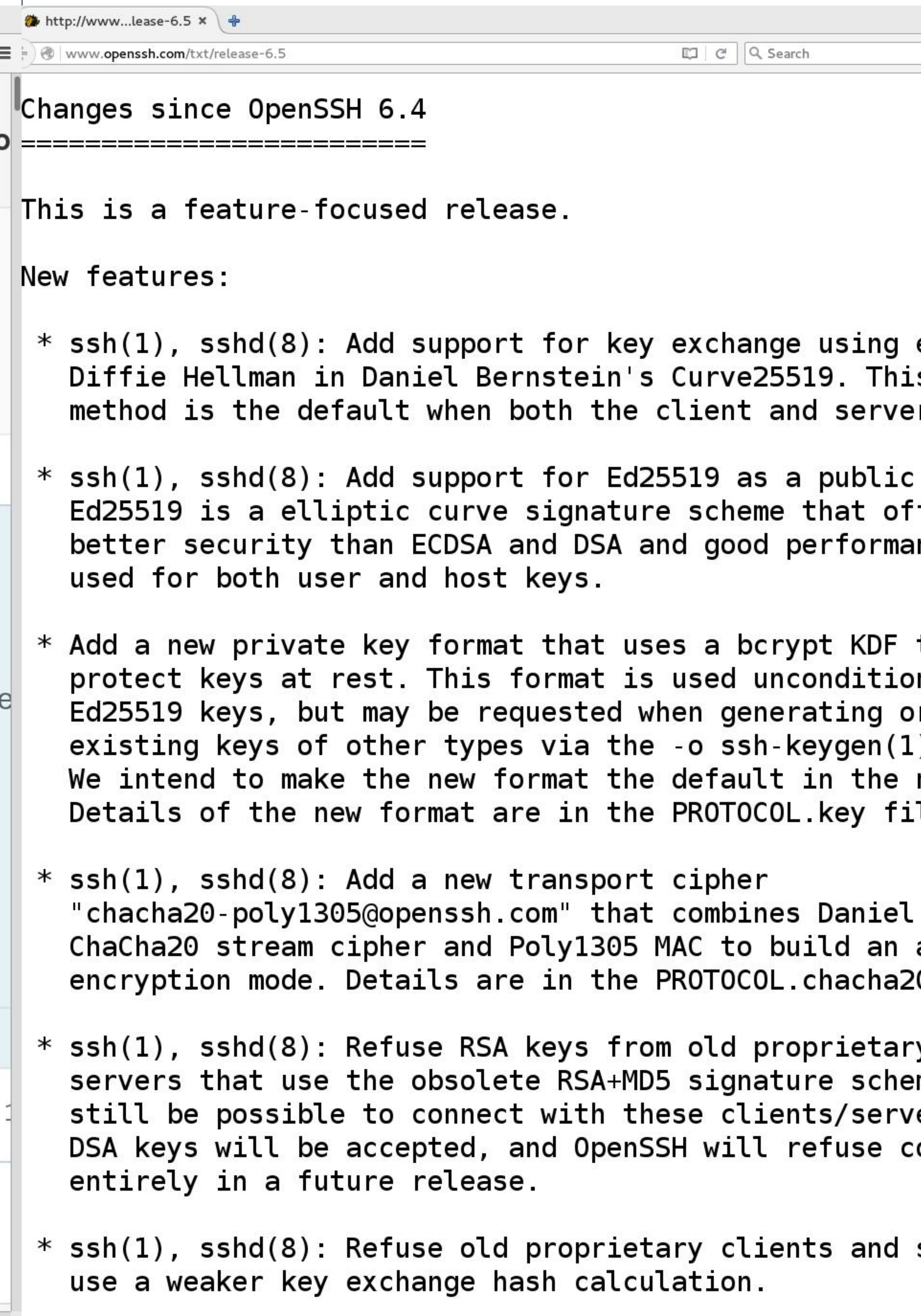
ID, and is kept
ise assigned to a
is class are stored
e wipe. If a file is
rm (as is all data



# 2013: Signal deploys Curve25519




# 2014: OpenSSH deploys Cu



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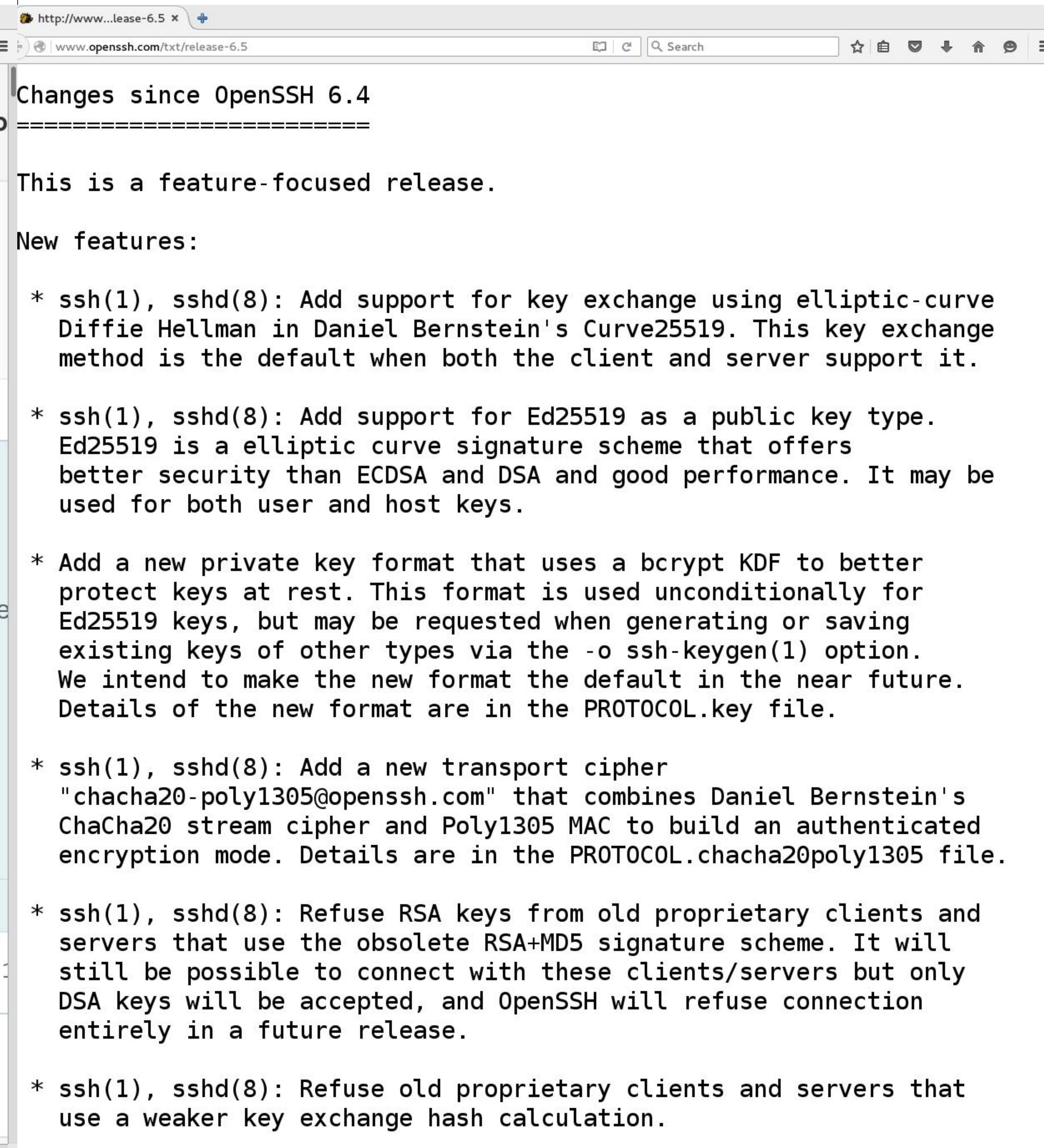

```
Changes since OpenSSH 6.4
=========================

This is a feature-focused release.

New features:

 * ssh(1), sshd(8): Add support for key exchange using elliptic-curve
   Diffie Hellman in Daniel Bernstein's Curve25519. This key exchange
   method is the default when both the client and server support it.

 * ssh(1), sshd(8): Add support for Ed25519 as a public key type.
   Ed25519 is a elliptic curve signature scheme that offers
   better security than ECDSA and DSA and good performance. It may be
   used for both user and host keys.

 * Add a new private key format that uses a bcrypt KDF to better
   protect keys at rest. This format is used unconditionally for
   Ed25519 keys, but may be requested when generating or saving
   existing keys of other types via the -o ssh-keygen(1) option.
   We intend to make the new format the default in the near future.
   Details of the new format are in the PROTOCOL.key file.

 * ssh(1), sshd(8): Add a new transport cipher
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 * ssh(1), sshd(8): Refuse old proprietary clients and servers that
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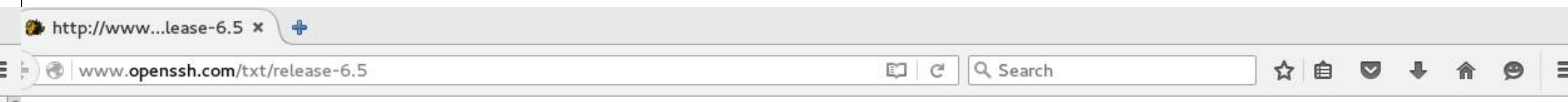


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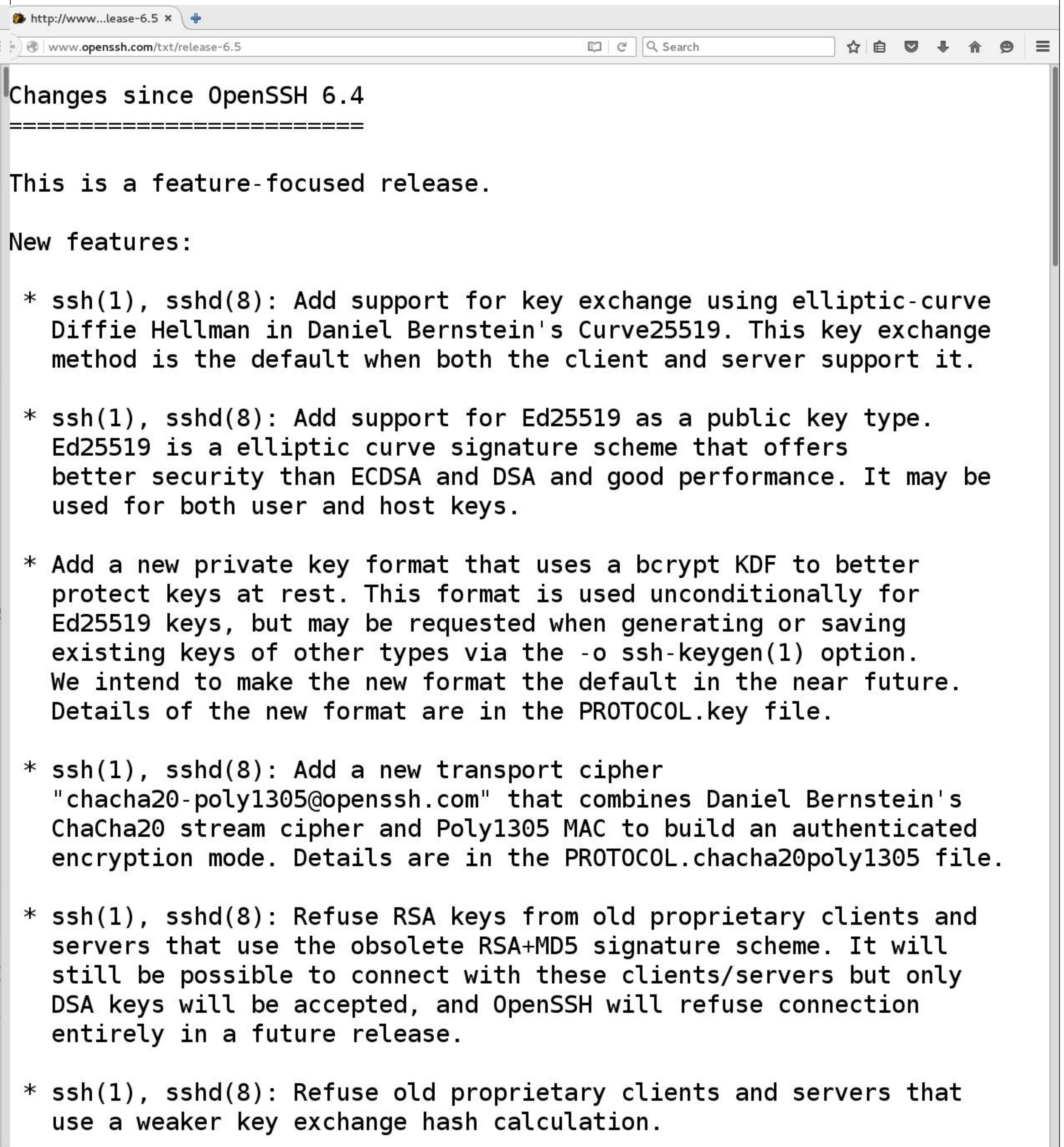


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2015.10: IRTF CF
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for signatures. Alr
X25519 and X448

2015.10: NIST re
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2015.11: BoringSS
X25519 and Ed25

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Many more: iani
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and /ed25519-de

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## 2014: OpenSSH deploys Curve25519

http://www...lease-6.5 ×

www.openssh.com/txt/release-6.5

Search

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These are just some highligh
Many more: ianix.com/pu
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2015.10: IRTF CFRG settles on EdDSA—Ed25519 and Ed448—for signatures. Already selected X25519 and X448 for DH.

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2015.11: BoringSSL adds X25519 and Ed25519.

These are just some highlights. Many more: `ianix.com/pub/curve25519-deployment.html` and `/ed25519-deployment.html`.