

Post-quantum RSA (pqRSA)

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Parameters

Scaled-down targets for cryptanalysis:

- ▶ pqr_{rsa}15: 2^{15} -byte keys using 512-bit primes.
- ▶ pqr_{rsa}20: 2^{20} -byte keys using 512-bit primes.
- ▶ pqr_{rsa}25: 2^{25} -byte keys using 1024-bit primes.

Primary parameter set included in submission:

- ▶ pqr_{rsa}30: 2^{30} -byte keys using 1024-bit primes.

Feasible option not included in submission:

- ▶ pqr_{rsa}40: 2^{40} -byte keys using 4096-bit primes.
Yes, we generated one of these keys.

Speeds

Approximate cycles/byte on 1 core of 3GHz Intel Skylake:

	keygen	dec	enc
pqrsa15	110000	3700	530
pqrsa20	110000	5800	1000
pqrsa25	540000	15000	1400
pqrsa30	550000	21000	1700

(Expect future speedups,
especially for keygen.)

pqrsa30 keygen: 2.3 days; dec: 2.1 hours; enc: 10.1 minutes.

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Submission also says "... quadrillion cycles".

Should say "trillion". NIST didn't notice?

Network traffic

For pqrsa30:

- ▶ Key: 2^{30} bytes.
- ▶ Signature: $\approx 2^{30}$ bytes.
- ▶ Ciphertext for kem: 2^{30} bytes.
- ▶ Ciphertext for encrypt: 2^{30} bytes,
including $\approx 2^{30}$ bytes of encrypted message.

Submission does not cover options for compressing signed messages.

Security against known attacks

pqrsa30 security analysis in submission:

- ▶ 2017 Häner–Roetteler–Svore \Rightarrow
 $\approx 2^{110}$ Toffoli gates using $\approx 2^{34}$ qubits.
Beyond NIST Category 2 under reasonable assumptions.

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Submitted to NIST as Category 2.

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RSA-2048 publicly broken by quantum computers:

“Yeah, NSA already told us to use RSA-3072.”

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Do users continue using RSA? Yes.

Analogy: “Lattice problems have been deeply studied by some of the great mathematicians going back to Gauss.”

Familiarity, continued: quotes from 1997

Lenstra: “The elliptic curve discrete logarithm problem has been around for a relatively short amount of time.”

Adleman: “I suspect that the lack of a sub-exponential algorithm is merely a matter of neglect.”

Schnorr: “It is unreasonable to assume that it has straight exponential complexity.”

Silverman: “Nor is it backed up by as many years of active cryptanalytic research as the RSA results are.”

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Analogy: If we say “Use 256-bit cipher keys”: Will users obey?

And is it clear that system X is better?

Maybe pqr30 is the strongest system in the NIST competition!