Lattice-based public-key cryptosystems

D. J. Bernstein

NIST post-quantum competition: 69 submissions in first round, from hundreds of people. (+13 submissions that NIST declared incomplete or improper.)

22 signature-system submissions. 5 lattice-based: Dilithium; DRS (broken); FALCON*; pqNTRUSign*; qTESLA.

47 encryption-system submissions. 20 lattice-based: Compact LWE* (broken); Ding*; EMBLEM; Frodo; HILA5 (CCA broken); KCL*; KINDI; Kyber; LAC; LIMA; Lizard*; LOTUS; NewHope; NTRUEncrypt; NTRU HRSS; NTRU Prime; Odd Manhattan; Round2*; SABER; Titanium.

*: submitter claims patent on this submission. Warning: even without *, submission could be covered by other patents!

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Let's try NTRU on the comp

Debian: `apt install sage`

Fedora: `yum install sager`

Source: www.sagemath.org

Web: sagecell.sagemath

Sage is Python 2
+ many math libraries
+ a few syntax differences:

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sage: 10^6 # power, not x
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sage: factor(31415926535
317213509 * 990371647
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```
sage: 10^6 # power, not xor
1000000
sage: factor(314159265358979323)
317213509 * 990371647
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sage: g
x^2 + 7*x + 2
sage: f+g    # built-in add
5*x^2 + 8*x + 5
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4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage:
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sage: f*(7*x)
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sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```

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x.<x> = ZZ[]
 now Zx is a class
 Zx objects are polys
 in x with int coeffs
 = Zx([3,1,4])

 x + 3
 = Zx([2,7,1])

*x + 2
+g    # built-in add
 8*x + 5
```



```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```

```
sage: #
sage: #
sage: de
....:
....:
sage:
```

```
Z[]
s a class
ts are polys
h int coeffs
1,4])

7,1])

uilt-in add
```



```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```

```
sage: # replace
sage: # x^(n+1)
sage: def convol
....:   return (
....:
sage:
```

```
s
lys
ffs
dd
```



```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```



```
sage: # replace x^n with
sage: # x^(n+1) with x, e
sage: def convolution(f,g
....:   return (f*g) % (x
....:
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```



```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:   return (f*g) % (x^n-1)
....:
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
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```



```
sage: # replace x^n with 1,
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sage: def convolution(f,g):
....:   return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
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 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```



```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:   return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
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....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
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4*x^4 + x^3 + 3*x^2
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True
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```



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sage: # replace x^n with 1,
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sage: def convolution(f,g):
....:   return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
```

```
*x   # built-in mul
 x^2 + 3*x
*x^2
 x^3 + 3*x^2
*2
 2*x + 6
*(7*x)
+ 7*x^2 + 21*x
*g
 29*x^3 + 18*x^2 + 23*x

*g == f*2+f*(7*x)+f*x^2
```



```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:   return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
```

```
sage: d
....:
....:
....:
....:
sage:
```

```
ilt-in mul
x

x^2



21*x

18*x^2 + 23*x

+f*(7*x)+f*x^2
```



```
sage: # replace x^n with 1,
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sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
```

```
sage: def random
....:   f = list
....:     for j
....:   return Z
....:
sage:
```

```
l
 23*x
f*x^2
```



```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:   return (f*g) % (x^n-1)
....:
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```
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....:   f = list(randrang
....:     for j in range(
....:   return Zx(f)
....:
sage:
```

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....:
sage: n = 7
sage:
```

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....:   return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage:
```

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....:   return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage:
```

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-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```

```
 replace x^n with 1,
 x^(n+1) with x, etc.
ef convolution(f,g):
 return (f*g) % (x^n-1)

 = 3  # global variable
onvolution(f,x)
*x + 4
onvolution(f,x^2)
 4*x + 1
onvolution(f,g)
+ 27*x + 35
```



```
sage: def randompoly():
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....:   return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```

Will use

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in submi

$n = 701$
$n = 743$
$n = 761$

```
x^n with 1,
with x, etc.
ution(f,g):
f*g) % (x^n-1)

lobal variable
n(f,x)

n(f,x^2)

n(f,g)
35
```



```
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-x^3 - x^2 - x - 1
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x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```

Will use bigger n f

Some choices of n
in submissions to

$n = 701$ for NTRU
$n = 743$ for NTRU
$n = 761$ for sntru

```
1,
etc.
):
^n-1)

riable
```



```
sage: def randompoly():
....:   f = list(randrange(3)-1
....:     for j in range(n))
....:   return Zx(f)
....:
sage: n = 7
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-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
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 x + 1
sage:
```

Will use bigger n for security

Some choices of n
in submissions to NIST:

$n = 701$ for NTRU HRSS.
$n = 743$ for NTRUEncrypt.
$n = 761$ for `sntrup4591761`

```
sage: def randompoly():
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....:     for j in range(n))
....:   return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
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```


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....:   return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
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```


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Overkill against attack algorithms known today, even for future attacker with quantum computer.

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sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
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sage:
```


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Can we find better algorithms?

```
sage: def randompoly():
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....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
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 x + 1
sage:
```


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Can we find better algorithms?

1998 NTRU paper took $n = 503$.

```
ef randompoly():
 f = list(randrange(3)-1
   for j in range(n))
 return Zx(f)

 = 7
andompoly()
x^2 - x - 1
andompoly()
^5 + x^3 - x
andompoly()
x^5 + x^4 - x^3 - x^2 +
```


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Modular

For integ
Sage's "
outputs

Matches

```
poly():
(randrange(3)-1
in range(n))
x(f)

()
 1
()
- x
()
 - x^3 - x^2 +
```


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Modular reduction

For integers `u`, `q` w
Sage's "`u%q`" alwa
outputs between 0

Matches standard

```
ge(3)-1
n))
x^2 +
```


Will use bigger n for security.

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Modular reduction

For integers `u`, `q` with `q` > 0
Sage's "`u%q`" always produc
outputs between 0 and `q` $-$

Matches standard math defi

Will use bigger n for security.

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in submissions to NIST:

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Overkill against attack algorithms known today, even for future attacker with quantum computer.

Can we find better algorithms?

1998 NTRU paper took $n = 503$.

Modular reduction

For integers `u`, `q` with $\mathtt{q} > 0$, Sage's "`u%q`" always produces outputs between 0 and $\mathtt{q} - 1$.

Matches standard math definition.

Will use bigger n for security.

Some choices of n
in submissions to NIST:

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Overkill against attack algorithms known today, even for future attacker with quantum computer.

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Modular reduction

For integers `u`, `q` with `q` > 0, Sage's "`u%q`" always produces outputs between 0 and `q` $- 1$.

Matches standard math definition.

Warning: Typically
`u` < 0 produces `u%q` < 0
in lower-level languages, so nonzero output leaks input sign.

Will use bigger n for security.

Some choices of n
in submissions to NIST:

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Modular reduction

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Warning: For polynomials `u`,
Sage can make the same mistake.

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issions to NIST:

for NTRU HRSS.
for NTRUEncrypt.
for `sntrup4591761`.

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with quantum computer.

find better algorithms?

TRU paper took $n = 503$.

Modular reduction

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Sage's "`u%q`" always produces
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Sage can make the same mistake.

```
sage: d
sage:
sage:
sage:
sage:
sage:
```

for security.

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r took $n = 503$.

Modular reduction

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Sage can make the same mistake.

```
sage: def balanc
sage:   g=list((
sage:   -q//2 fo
sage:   return Z
sage:
sage:
```

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Modular reduction

For integers u, q with $q > 0$,
Sage's "u%q" always produces
outputs between 0 and $q - 1$.

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in lower-level languages, so
nonzero output leaks input sign.

Warning: For polynomials u,
Sage can make the same mistake.

```
sage: def balancedmod(f,q
sage:    g=list(((f[i]+q//
sage:    -q//2 for i in ra
sage:    return Zx(g)
sage:
sage:
```

Modular reduction

For integers u, q with $q > 0$, Sage's "u%q" always produces outputs between 0 and $q - 1$.

Matches standard math definition.

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Warning: For polynomials u, Sage can make the same mistake.


```
sage: def balancedmod(f,q):
sage:   g=list(((f[i]+q//2)%q)
sage:   -q//2 for i in range(n))
sage:   return Zx(g)
sage:
sage:
```

Modular reduction

For integers u, q with $q > 0$, Sage's "u%q" always produces outputs between 0 and $q - 1$.

Matches standard math definition.

Warning: Typically $u < 0$ produces $u\%q < 0$ in lower-level languages, so nonzero output leaks input sign.

Warning: For polynomials u, Sage can make the same mistake.


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sage: def balancedmod(f,q):
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```

```
sage: def invert
....:   Fp = Int
....:   Fpx = Zx
....:   T = Fpx.
....:   return Z
....:
sage:
```

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```
sage: def invertmodprime(
....:   Fp = Integers(p)
....:   Fpx = Zx.change_r
....:   T = Fpx.quotient(
....:   return Zx(lift(1/
....:
sage:
```

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```

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....:   return Zx(lift(1/T(f)))
....:
sage: n = 7
sage: f = randompoly()
sage: f3 = invertmodprime(f,3)
sage:
```

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sage: convolution(f,f3)
6*x^6 + 6*x^5 + 3*x^4 + 3*x^3 +
 3*x^2 + 3*x + 4
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```

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6*x^6 + 6*x^5 + 3*x^4 + 3*x^3 +
 3*x^2 + 3*x + 4
sage:
```

```
def inve
  assert
  g = in
  M = ba
  C = co
  while
    r =
    if i
    g =
```

Exercise

invertm

Hint: Co

```
edmod(f,q):
(f[i]+q//2)%q)
r i in range(n))
x(g)

9*x



% 200

d(u,200)
```



```
sage: def invertmodprime(f,p):
....:   Fp = Integers(p)
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sage: n = 7
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6*x^6 + 6*x^5 + 3*x^4 + 3*x^3 +
 3*x^2 + 3*x + 4
sage:
```

```
def invertmodpow
  assert q.is_po
  g = invertmodp
  M = balancedmo
  C = convolutio
  while True:
    r = M(C(g,f)
    if r == 1: r
    g = M(C(g,2-
```

Exercise: Figure o `invertmodpowerc`

Hint: Compare `r`

```
):
2)%q)
nge(n))
```



```
sage: def invertmodprime(f,p):
....:   Fp = Integers(p)
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....:   T = Fpx.quotient(x^n-1)
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sage: n = 7
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6*x^6 + 6*x^5 + 3*x^4 + 3*x^3 +
 3*x^2 + 3*x + 4
sage:
```

```
def invertmodpowerof2(f,q
  assert q.is_power_of(2)
  g = invertmodprime(f,2)
  M = balancedmod
  C = convolution
  while True:
    r = M(C(g,f),q)
    if r == 1: return g
    g = M(C(g,2-r),q)
```

Exercise: Figure out how `invertmodpowerof2` works.

Hint: Compare `r` to previous

```
sage: def invertmodprime(f,p):
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 = 7
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+ 3*x + 4
```



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sage: n
sage: q
sage:
```

```
modprime(f,p):
egers(p)
.change_ring(Fp)
quotient(x^n-1)
x(lift(1/T(f)))


poly()
tmodprime(f,3)
n(f,f3)
3*x^4 + 3*x^3 +
```



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Hint: Compare `r` to previous `r`.

```
sage: n = 7
sage: q = 256
sage:
```

```
f,p):

ring(Fp)
(x^n-1)
T(f)))

(f,3)

3*x^3 +
```



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sage: convolution(f,g)
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sage:
```

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1
sage:
```

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n, positi
q, power

```
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rime(f,2)
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,q)
eturn g
r),q)
```

ut how
of2 works.
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1
sage:
```

NTRU key genera

Parameters:

n, positive integer

q, power of 2 (e.g

):

.

s r.


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sage: balancedmod(_,q)
1
sage:
```

NTRU key generation

Parameters:

n, positive integer (e.g., 701

q, power of 2 (e.g., 4096).

```
sage: n = 7
sage: q = 256
sage: f = randompoly()
sage: f
-x^6 - x^4 + x^2 + x - 1
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1
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```


NTRU key generation

Parameters:

n, positive integer (e.g., 701);

q, power of 2 (e.g., 4096).

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sage: f = randompoly()
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sage: balancedmod(_,q)
1
sage:
```


NTRU key generation

Parameters:

n, positive integer (e.g., 701);

q, power of 2 (e.g., 4096).

Secret key:

random n-coeff polynomial a;

random n-coeff polynomial d;

all coefficients in $\{-1, 0, 1\}$.

```
sage: n = 7
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NTRU key generation

Parameters:
n, positive integer (e.g., 701);
q, power of 2 (e.g., 4096).

Secret key:
random n-coeff polynomial a;
random n-coeff polynomial d;
all coefficients in $\{-1, 0, 1\}$.

Require d invertible mod q.
Require d invertible mod 3.

```
sage: n = 7
sage: q = 256
sage: f = randompoly()
sage: f
-x^6 - x^4 + x^2 + x - 1
sage: g = invertmodpowerof2(f,q)
sage: g
47*x^6 + 126*x^5 - 54*x^4 -
 87*x^3 - 36*x^2 - 58*x + 61
sage: convolution(f,g)
-256*x^5 - 256*x^4 + 256*x + 257
sage: balancedmod(_,q)
1
sage:
```


NTRU key generation

Parameters:
n, positive integer (e.g., 701);
q, power of 2 (e.g., 4096).

Secret key:
random n-coeff polynomial a;
random n-coeff polynomial d;
all coefficients in $\{-1, 0, 1\}$.

Require d invertible mod q.
Require d invertible mod 3.

Public key: $A = 3a/d$ in the ring $R_q = (\mathbf{Z}/q)[x]/(x^n - 1)$.

```
 = 7
 = 256
 = randompoly()

x^4 + x^2 + x - 1
 = invertmodpowerof2(f,q)

+ 126*x^5 - 54*x^4 -
 - 36*x^2 - 58*x + 61
onvolution(f,g)
5 - 256*x^4 + 256*x + 257
alancedmod(_,q)
```


NTRU key generation

Parameters:
n, positive integer (e.g., 701);
q, power of 2 (e.g., 4096).

Secret key:
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Require d invertible mod q.
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Public key: $A = 3a/d$ in the ring $R_q = (\mathbf{Z}/q)[x]/(x^n - 1)$.

```
def key
  while
    try
      d
      d3
      d
      b
    exc
      pa
  a = r
  publi

  secre
  retur
```

```
poly()

 + x - 1
modpowerof2(f,q)

 - 54*x^4 -
 - 58*x + 61
n(f,g)
^4 + 256*x + 257
d(_,q)
```


NTRU key generation

Parameters:
n, positive integer (e.g., 701);
q, power of 2 (e.g., 4096).

Secret key:
random n-coeff polynomial a;
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all coefficients in $\{-1, 0, 1\}$.

Require d invertible mod q.
Require d invertible mod 3.

Public key: $A = 3a/d$ in the ring
$R_q = (\mathbf{Z}/q)[x]/(x^n - 1)$.

```
def keypair():
  while True:
    try:
      d = random
      d3 = inver
      dq = inver
      break
    except:
      pass
  a = randompoly
  publickey = ba
              con
  secretkey = d,
  return publick
```

```
f2(f,q)

 -
 61

x + 257
```


NTRU key generation

Parameters:

n, positive integer (e.g., 701);

q, power of 2 (e.g., 4096).

Secret key:

random n-coeff polynomial a;

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all coefficients in $\{-1, 0, 1\}$.

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Public key: $A = 3a/d$ in the ring $R_q = (\mathbf{Z}/q)[x]/(x^n - 1)$.


```
def keypair():
  while True:
    try:
      d = randompoly()
      d3 = invertmodprime
      dq = invertmodpower
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod
              convolution(
  secretkey = d,d3
  return publickey,secret
```

NTRU key generation

Parameters:

n, positive integer (e.g., 701);

q, power of 2 (e.g., 4096).

Secret key:

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```
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    try:
      d = randompoly()
      d3 = invertmodprime(d,3)
      dq = invertmodpowerof2(d,q)
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
                  convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```

key generation

ers:

ve integer (e.g., 701);

r of 2 (e.g., 4096).

ey:

n-coeff polynomial a;

n-coeff polynomial d;

icients in $\{-1, 0, 1\}$.

d invertible mod q.

d invertible mod 3.

ey: $A = 3a/d$ in the ring

$\mathbb{Z}/q)[x]/(x^n - 1)$.


```
def keypair():
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    try:
      d = randompoly()
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      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
                  convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```

```
sage: A
sage:
```

tion

(e.g., 701);
., 4096).

lynomial a;
lynomial d;
$\{-1, 0, 1\}$.

le mod q.
le mod 3.

$3a/d$ in the ring
$^n - 1)$.


```
def keypair():
  while True:
    try:
      d = randompoly()
      d3 = invertmodprime(d,3)
      dq = invertmodpowerof2(d,q)
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
                convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```

```
sage: A,secretke
sage:
```

);

a;

d;

e ring


```
def keypair():
  while True:
    try:
      d = randompoly()
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      dq = invertmodpowerof2(d,q)
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
                convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```

```
sage: A,secretkey = keypa
sage:
```

```
def keypair():
  while True:
    try:
      d = randompoly()
      d3 = invertmodprime(d,3)
      dq = invertmodpowerof2(d,q)
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
              convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```



```
sage: A,secretkey = keypair()
sage:
```

```
def keypair():
  while True:
    try:
      d = randompoly()
      d3 = invertmodprime(d,3)
      dq = invertmodpowerof2(d,q)
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
              convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage:
```

```
def keypair():
  while True:
    try:
      d = randompoly()
      d3 = invertmodprime(d,3)
      dq = invertmodpowerof2(d,q)
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
              convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```



```
sage: A,secretkey = keypair()
sage: A
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 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage:
```

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      dq = invertmodpowerof2(d,q)
      break
    except:
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  a = randompoly()
  publickey = balancedmod(3 *
              convolution(a,dq),q)
  secretkey = d,d3
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```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage:
```

```
def keypair():
  while True:
    try:
      d = randompoly()
      d3 = invertmodprime(d,3)
      dq = invertmodpowerof2(d,q)
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
              convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage:
```

```
def keypair():
  while True:
    try:
      d = randompoly()
      d3 = invertmodprime(d,3)
      dq = invertmodpowerof2(d,q)
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
                convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```

```
pair():
 True:
:
 = randompoly()
3 = invertmodprime(d,3)
q = invertmodpowerof2(d,q)
reak
ept:
ass
andompoly()
ckey = balancedmod(3 *
       convolution(a,dq),q)
tkey = d,d3
n publickey,secretkey
```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```

NTRU e

One mo

w, posit

```
poly()
tmodprime(d,3)
tmodpowerof2(d,q)

()
lancedmod(3 *
volution(a,dq),q)
d3
ey,secretkey
```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```

NTRU encryption

One more paramet

w, positive intege

```
e(d,3)
rof2(d,q)
l(3 *
(a,dq),q)
key
```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```

NTRU encryption

One more parameter:

w, positive integer (e.g., 46

```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```


NTRU encryption

One more parameter:
w, positive integer (e.g., 467).

```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```


NTRU encryption

One more parameter:
w, positive integer (e.g., 467).

Message for encryption:
n-coeff weight-w polynomial c
with all coeffs in $\{-1, 0, 1\}$.

"Weight w": w nonzero coeffs,
$n - w$ zero coeffs.

```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```


NTRU encryption

One more parameter:
w, positive integer (e.g., 467).

Message for encryption:
n-coeff weight-w polynomial c
with all coeffs in $\{-1, 0, 1\}$.

"Weight w": w nonzero coeffs,
$n - w$ zero coeffs.

Ciphertext: $C = Ab + c$ in R_q
where b is chosen randomly
from the set of messages.

```
,secretkey = keypair()

6 - 31*x^5 - 118*x^4 -
 + 73*x^2 - 16*x + 7
,d3 = secretkey

x^5 - x^4 + x^3 - 1
onvolution(d,A)
+ 253*x^5 + 253*x^3 -
2 - 3*x - 3
alancedmod(_,q)
- 3*x^5 - 3*x^3 + 3*x^2
- 3
```


NTRU encryption

One more parameter:
w, positive integer (e.g., 467).

Message for encryption:
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Ciphertext: $C = Ab + c$ in R_q
where b is chosen randomly
from the set of messages.

```
sage: d
....:
....:
....:
....:
....:
....:
....:
....:
....:
....:
sage: w
sage: ra
-x^6 - 
sage:
```

```
y = keypair()

5 - 118*x^4 -
 - 16*x + 7
retkey

 + x^3 - 1
n(d,A)
 + 253*x^3 -
 3
d(_,q)
 3*x^3 + 3*x^2
```


NTRU encryption

One more parameter:
w, positive integer (e.g., 467).

Message for encryption:
n-coeff weight-w polynomial c
with all coeffs in $\{-1, 0, 1\}$.

"Weight w": w nonzero coeffs,
$n - w$ zero coeffs.

Ciphertext: $C = Ab + c$ in R_q
where b is chosen randomly
from the set of messages.

```
sage: def random
....:   R = rand
....:   assert w
....:   c = n*[0
....:   for j in
....:     while
....:       r =
....:       if n
....:     c[r] =
....:   return Z
....:
sage: w = 5
sage: randommess
-x^6 - x^5 + x^4
sage:
```

```
ir()
^4 -
7
1
3 -
3*x^2
```


NTRU encryption

One more parameter:
w, positive integer (e.g., 467).

Message for encryption:
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Ciphertext: $C = Ab + c$ in R_q where b is chosen randomly from the set of messages.

```
sage: def randommessage()
....:   R = randrange
....:   assert w <= n
....:   c = n*[0]
....:   for j in range(w)
....:     while True:
....:       r = R(n)
....:       if not c[r]:
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 -
sage:
```

NTRU encryption

One more parameter:
w, positive integer (e.g., 467).

Message for encryption:
n-coeff weight-w polynomial c
with all coeffs in $\{-1, 0, 1\}$.

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....:   c = n*[0]
....:   for j in range(w):
....:     while True:
....:       r = R(n)
....:       if not c[r]: break
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```

encryption

re parameter:
ive integer (e.g., 467).

for encryption:
weight-w polynomial c
coeffs in $\{-1, 0, 1\}$.

w": w nonzero coeffs,
ero coeffs.

ext: $C = Ab + c$ in R_q
is chosen randomly
set of messages.


```
sage: def randommessage():
....:   R = randrange
....:   assert w <= n
....:   c = n*[0]
....:   for j in range(w):
....:     while True:
....:       r = R(n)
....:       if not c[r]: break
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```

```
sage: de
....:
....:
....:
....:
....:
sage:
```

ter:

r (e.g., 467).

ption:

polynomial c

$\{-1, 0, 1\}$.

onzero coeffs,

$Ab + c$ in R_q

randomly

essages.


```
sage: def randommessage():
....:   R = randrange
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....:   c = n*[0]
....:   for j in range(w):
....:     while True:
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....:       if not c[r]: break
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```

```
sage: def encryp
....:   b = rand
....:   Ab = con
....:   C = bala
....:   return C
....:
sage:
```

7).

l c

effs,

R_q


```
sage: def randommessage():
....:   R = randrange
....:   assert w <= n
....:   c = n*[0]
....:   for j in range(w):
....:     while True:
....:       r = R(n)
....:       if not c[r]: break
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```



```
sage: def encrypt(c,A):
....:   b = randommessage
....:   Ab = convolution(
....:   C = balancedmod(A
....:   return C
....:
sage:
```

```
sage: def randommessage():
....:   R = randrange
....:   assert w <= n
....:   c = n*[0]
....:   for j in range(w):
....:     while True:
....:       r = R(n)
....:       if not c[r]: break
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```



```
sage: def encrypt(c,A):
....:   b = randommessage()
....:   Ab = convolution(A,b)
....:   C = balancedmod(Ab + c,q)
....:   return C
....:
sage:
```

```
sage: def randommessage():
....:   R = randrange
....:   assert w <= n
....:   c = n*[0]
....:   for j in range(w):
....:     while True:
....:       r = R(n)
....:       if not c[r]: break
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```



```
sage: def encrypt(c,A):
....:   b = randommessage()
....:   Ab = convolution(A,b)
....:   C = balancedmod(Ab + c,q)
....:   return C
....:
sage: A,secretkey = keypair()
sage:
```

```
sage: def randommessage():
....:   R = randrange
....:   assert w <= n
....:   c = n*[0]
....:   for j in range(w):
....:     while True:
....:       r = R(n)
....:       if not c[r]: break
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
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sage:
```



```
sage: def encrypt(c,A):
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....:   return C
....:
sage: A,secretkey = keypair()
sage: c = randommessage()
sage:
```

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sage: def randommessage():
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....:   for j in range(w):
....:     while True:
....:       r = R(n)
....:       if not c[r]: break
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```



```
sage: def encrypt(c,A):
....:   b = randommessage()
....:   Ab = convolution(A,b)
....:   C = balancedmod(Ab + c,q)
....:   return C
....:
sage: A,secretkey = keypair()
sage: c = randommessage()
sage: C = encrypt(c,A)
sage:
```

```
sage: def randommessage():
....:   R = randrange
....:   assert w <= n
....:   c = n*[0]
....:   for j in range(w):
....:     while True:
....:       r = R(n)
....:       if not c[r]: break
....:     c[r] = 1-2*R(2)
....:   return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```



```
sage: def encrypt(c,A):
....:   b = randommessage()
....:   Ab = convolution(A,b)
....:   C = balancedmod(Ab + c,q)
....:   return C
....:
sage: A,secretkey = keypair()
sage: c = randommessage()
sage: C = encrypt(c,A)
sage: C
21*x^6 - 48*x^5 + 31*x^4 -
 76*x^3 - 77*x^2 + 15*x - 113
sage:
```

```
ef randommessage():
 R = randrange
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```

NTRU d

Comput

```
message():
range
 <= n
]
 range(w):
True:
R(n)
ot c[r]: break
 1-2*R(2)
x(c)


age()
 + x^3 - x^2
```



```
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```

NTRU decryption

Compute $dC = 3a$

```
:

:

break

x^2
```



```
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NTRU decryption

Compute $dC = 3ab + dc$ in

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```
ef encrypt(c,A):
 b = randommessage()
 Ab = convolution(A,b)
 C = balancedmod(Ab + c,q)
 return C

,secretkey = keypair()
 = randommessage()
 = encrypt(c,A)

- 48*x^5 + 31*x^4 -
 - 77*x^2 + 15*x - 113
```


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```
sage: d
....:
....:
....:
....:
....:
....:
sage:
```

```
t(c,A):
ommessage()
volution(A,b)
ncedmod(Ab + c,q)


y = keypair()
message()
t(c,A)


+ 31*x^4 -
 + 15*x - 113
```


NTRU decryption

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```
sage: def decryp
....:     M = ba
....:     f,r =
....:     u=M(co
....:     c=M(co
....:     return
....:
sage:
```

```
()
(A,b)
b + c,q)

ir()

-
113
```

NTRU decryption

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```
sage: def decrypt(C,secre
....:     M = balancedmod
....:     f,r = secretkey
....:     u=M(convolution
....:     c=M(convolution
....:     return c
....:
sage:
```

NTRU decryption

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```
sage: def decrypt(C,secretkey):
....:     M = balancedmod
....:     f,r = secretkey
....:     u=M(convolution(C,f),q)
....:     c=M(convolution(u,r),3)
....:     return c
....:
sage:
```

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....:
sage: c
x^5 + x^4 - x^3 + x + 1
sage:
```

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```

lecryption

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' have small coeffs,
- dc is not very big.
e that coeffs of $3ab + dc$
veen $-q/2$ and $q/2 - 1$.

$b + dc$ in R_q reveals
c in $R = \mathbf{Z}[x]/(x^n - 1)$.
modulo 3: dc in R_3.

by $1/d$ in R_3
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```

```
sage: n
sage: w
sage: q
sage:
```

$ab + dc$ in R_q.

ll coeffs,
t very big.
ffs of $3ab + dc$
and $q/2 - 1$.

R_q reveals
$\mathbf{Z}[x]/(x^n - 1)$.
dc in R_3.

n R_3
e c in R_3.
n -1 and 1,


```
sage: def decrypt(C,secretkey):
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sage: c
x^5 + x^4 - x^3 + x + 1
sage: decrypt(C,secretkey)
x^5 + x^4 - x^3 + x + 1
sage:
```

```
sage: n = 7
sage: w = 5
sage: q = 256
sage:
```

R_q.

$+ dc$

$- 1$.

s

$- 1)$.

.

1,


```
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```

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sage: n = 7
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sage: A,secretkey = keypair()
sage:
```

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x^5 + x^4 - x^3 + x + 1
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```



```
sage: n = 7
sage: w = 5
sage: q = 256
sage: A,secretkey = keypair()
sage: A
-101*x^6 - 76*x^5 - 90*x^4 -
 83*x^3 + 40*x^2 + 108*x - 54
sage:
```

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sage: def decrypt(C,secretkey):
....:     M = balancedmod
....:     f,r = secretkey
....:     u=M(convolution(C,f),q)
....:     c=M(convolution(u,r),3)
....:     return c
....:
sage: c
x^5 + x^4 - x^3 + x + 1
sage: decrypt(C,secretkey)
x^5 + x^4 - x^3 + x + 1
sage:
```



```
sage: n = 7
sage: w = 5
sage: q = 256
sage: A,secretkey = keypair()
sage: A
-101*x^6 - 76*x^5 - 90*x^4 -
 83*x^3 + 40*x^2 + 108*x - 54
sage: d,d3 = secretkey
sage:
```

```
sage: def decrypt(C,secretkey):
....:     M = balancedmod
....:     f,r = secretkey
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....:     c=M(convolution(u,r),3)
....:     return c
....:
sage: c
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x^5 + x^4 - x^3 + x + 1
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 83*x^3 + 40*x^2 + 108*x - 54
sage: d,d3 = secretkey
sage: d
x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage:
```

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x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage: M = balancedmod
sage:
```

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x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage: M = balancedmod
sage: a3 = M(conv(d,A),q)
sage:
```

```
sage: def decrypt(C,secretkey):
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sage: c
x^5 + x^4 - x^3 + x + 1
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```



```
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sage: d
x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage: M = balancedmod
sage: a3 = M(conv(d,A),q)
sage: a3
3*x^2 - 3*x
```

```
ef decrypt(C,secretkey):
   M = balancedmod
   f,r = secretkey
   u=M(convolution(C,f),q)
   c=M(convolution(u,r),3)
   return c
```

```
^4 - x^3 + x + 1
ecrypt(C,secretkey)
^4 - x^3 + x + 1
```



```
sage: n = 7
sage: w = 5
sage: q = 256
sage: A,secretkey = keypair()
sage: A
-101*x^6 - 76*x^5 - 90*x^4 -
 83*x^3 + 40*x^2 + 108*x - 54
sage: d,d3 = secretkey
sage: d
x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage: M = balancedmod
sage: a3 = M(conv(d,A),q)
sage: a3
3*x^2 - 3*x
```

```
sage: c
sage:
```

```
t(C,secretkey):
lancedmod
secretkey
nvolution(C,f),q)
nvolution(u,r),3)
 c


+ x + 1
secretkey)
+ x + 1
```



```
sage: n = 7
sage: w = 5
sage: q = 256
sage: A,secretkey = keypair()
sage: A
-101*x^6 - 76*x^5 - 90*x^4 -
 83*x^3 + 40*x^2 + 108*x - 54
sage: d,d3 = secretkey
sage: d
x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage: M = balancedmod
sage: a3 = M(conv(d,A),q)
sage: a3
3*x^2 - 3*x
```

```
sage: c = random
sage:
```

```
tkey):
.
-
.(C,f),q)
.(u,r),3)
-)
```



```
sage: n = 7
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sage: q = 256
sage: A,secretkey = keypair()
sage: A
-101*x^6 - 76*x^5 - 90*x^4 -
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```



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```



```
sage: c = randommessage()
sage: b = randommessage()
sage:
```

```
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x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
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sage: a3 = M(conv(d,A),q)
sage: a3
3*x^2 - 3*x
```



```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage:
```

```
sage: n = 7
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sage: A,secretkey = keypair()
sage: A
-101*x^6 - 76*x^5 - 90*x^4 -
 83*x^3 + 40*x^2 + 108*x - 54
sage: d,d3 = secretkey
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x^5 + x^4 - x^3 + x - 1
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3*x^2 - 3*x
```



```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage:
```

```
sage: n = 7
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sage: A,secretkey = keypair()
sage: A
-101*x^6 - 76*x^5 - 90*x^4 -
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x^5 + x^4 - x^3 + x - 1
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```



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 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage:
```

```
sage: n = 7
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-101*x^6 - 76*x^5 - 90*x^4 -
 83*x^3 + 40*x^2 + 108*x - 54
sage: d,d3 = secretkey
sage: d
x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage: M = balancedmod
sage: a3 = M(conv(d,A),q)
sage: a3
3*x^2 - 3*x
```



```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage:
```

```
sage: n = 7
sage: w = 5
sage: q = 256
sage: A,secretkey = keypair()
sage: A
-101*x^6 - 76*x^5 - 90*x^4 -
 83*x^3 + 40*x^2 + 108*x - 54
sage: d,d3 = secretkey
sage: d
x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage: M = balancedmod
sage: a3 = M(conv(d,A),q)
sage: a3
3*x^2 - 3*x
```



```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```

```
= 7
= 5
= 256
,secretkey = keypair()

6 - 76*x^5 - 90*x^4 -
 + 40*x^2 + 108*x - 54
,d3 = secretkey

^4 - x^3 + x - 1
onv = convolution
 = balancedmod
3 = M(conv(d,A),q)
3
 3*x
```



```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```

```
sage: M
x^6 - x
 + 1
sage:
```

```
y = keypair()

5 - 90*x^4 -
 + 108*x - 54
retkey

+ x - 1
volution
edmod
v(d,A),q)
```



```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```

```
sage: M(u,3)
x^6 - x^5 + x^4
 + 1
sage:
```

ir()

4 -

- 54


```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x
 + 1
sage:
```

```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage:
```

```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
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sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage:
```

```
sage: c = randommessage()
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sage: C = M(conv(A,b)+c,q)
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-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
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sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage:
```

```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage:
```

```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```

```
= randommessage()
= randommessage()
= M(conv(A,b)+c,q)

+ 28*x^5 + 114*x^4 +
- 37*x^2 + 16*x + 119
= M(conv(C,d),q)

+ 2*x^5 + 4*x^4 - x^3 -
+ 5*x + 1
onv(a3,b)+conv(c,d)
+ 2*x^5 + 4*x^4 - x^3 -
+ 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```

Does de

All coeff

All coeff

and exac

```
message()
message()
(A,b)+c,q)

 + 114*x^4 +
 + 16*x + 119
(C,d),q)

 4*x^4 - x^3 -

+conv(c,d)
 4*x^4 - x^3 -
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```

Does decryption a

All coeffs of a are
All coeffs of b are
and exactly w are

```
)
4 +
119
x^3 -
)
x^3 -
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```

Does decryption always work

All coeffs of a are in $\{-1, 0,$

All coeffs of b are in $\{-1, 0,$

and exactly w are nonzero.

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.
(Same argument would work for
b of any weight, a of weight w.)

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
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Each coeff of ab in R
has absolute value at most w.
(Same argument would work for
b of any weight, a of weight w.)

Similar comments for d, c.
Each coeff of $3ab + dc$ in R
has absolute value at most $4w$.

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.
(Same argument would work for
b of any weight, a of weight w.)

Similar comments for d, c.
Each coeff of $3ab + dc$ in R
has absolute value at most $4w$.

e.g. $w = 467$: at most 1868.
Decryption works for $q = 4096$.

```
(u,3)
^5 + x^4 - x^3 - x^2 - x

(conv(c,d),3)
^5 + x^4 - x^3 - x^2 - x

onv(M(u,3),d3)
^5 - x^4 - 3*x^3 - x^2 +

(_,3)
^5 - x^4 - x^2 + x

^5 - x^4 - x^2 + x
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$, and exactly w are nonzero.

Each coeff of ab in R has absolute value at most w.
(Same argument would work for b of any weight, a of weight w.)

Similar comments for d, c.
Each coeff of $3ab + dc$ in R has absolute value at most $4w$.

e.g. $w = 467$: at most 1868.
Decryption works for $q = 4096$.

What ab

- x^3 - x^2 - x

),3)

- x^3 - x^2 - x

),d3)

- 3*x^3 - x^2 +

- x^2 + x

- x^2 + x

Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.
(Same argument would work for
b of any weight, a of weight w.)

Similar comments for d, c.
Each coeff of $3ab + dc$ in R
has absolute value at most $4w$.

e.g. $w = 467$: at most 1868.
Decryption works for $q = 4096$.

What about $w =$

```
:^2 - x

:^2 - x

- x^2 +

:

:
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.
(Same argument would work for
b of any weight, a of weight w.)

Similar comments for d, c.
Each coeff of $3ab + dc$ in R
has absolute value at most $4w$.

e.g. $w = 467$: at most 1868.
Decryption works for $q = 4096$.

What about $w = 467$, $q = 2$

Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.
(Same argument would work for
b of any weight, a of weight w.)

Similar comments for d, c.
Each coeff of $3ab + dc$ in R
has absolute value at most $4w$.

e.g. $w = 467$: at most 1868.
Decryption works for $q = 4096$.

What about $w = 467$, $q = 2048$?

Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
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Each coeff of ab in R
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(Same argument would work for
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Similar comments for d, c.
Each coeff of $3ab + dc$ in R
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New Hope submission: $<2^{-213}$.

KINDI: 2^{-165}.

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NTRUEncrypt: $<2^{-80}$.

KCL: $\approx 2^{-60}$.

Ding: $\approx 2^{-60}$, only IND-CPA.

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so $a - (A/3)d_2 = (A/3)d_1$.
Eliminate a: almost certainly
$H(-(A/3)d_2) = H((A/3)d_1)$ for
$H(f) = ([f_0 < 0], \ldots, [f_{k-1} < 0])$.

Enumerate all $H(-(A/3)d_2)$.
Enumerate all $H((A/3)d_1)$.
Search for collisions.
Only about $3^{n/2}$ computations;
but beware cost of memory.

Lattices

This is a lettuce:

Collision attacks

Write d as $d_1 + d_2$ where
d_1 = bottom $\lceil n/2 \rceil$ terms of d,
d_2 = remaining terms of d.

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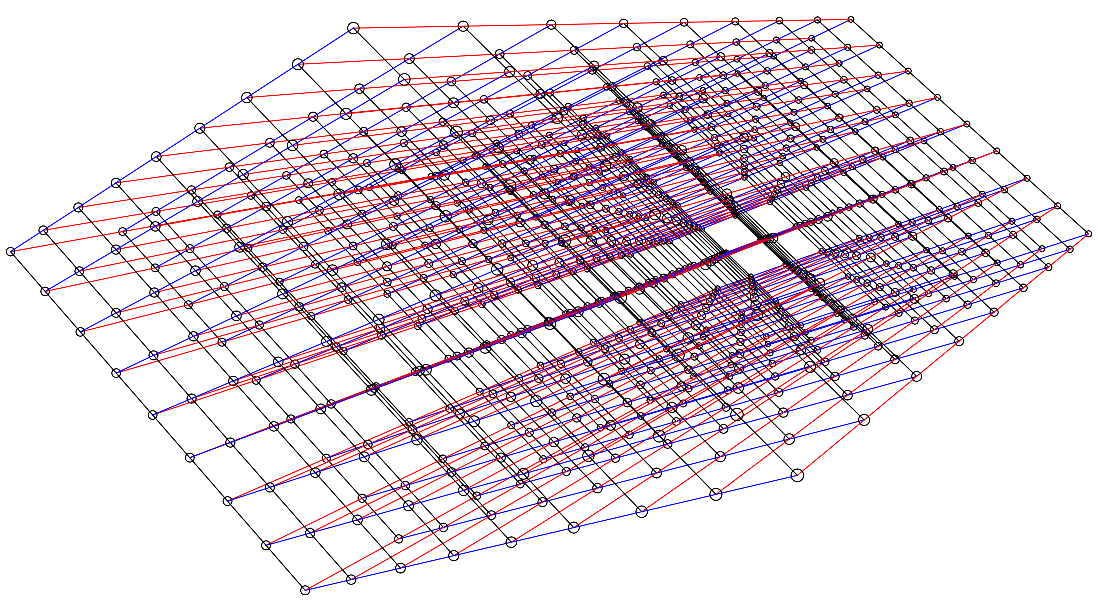

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ttom $\lceil n/2 \rceil$ terms of d,
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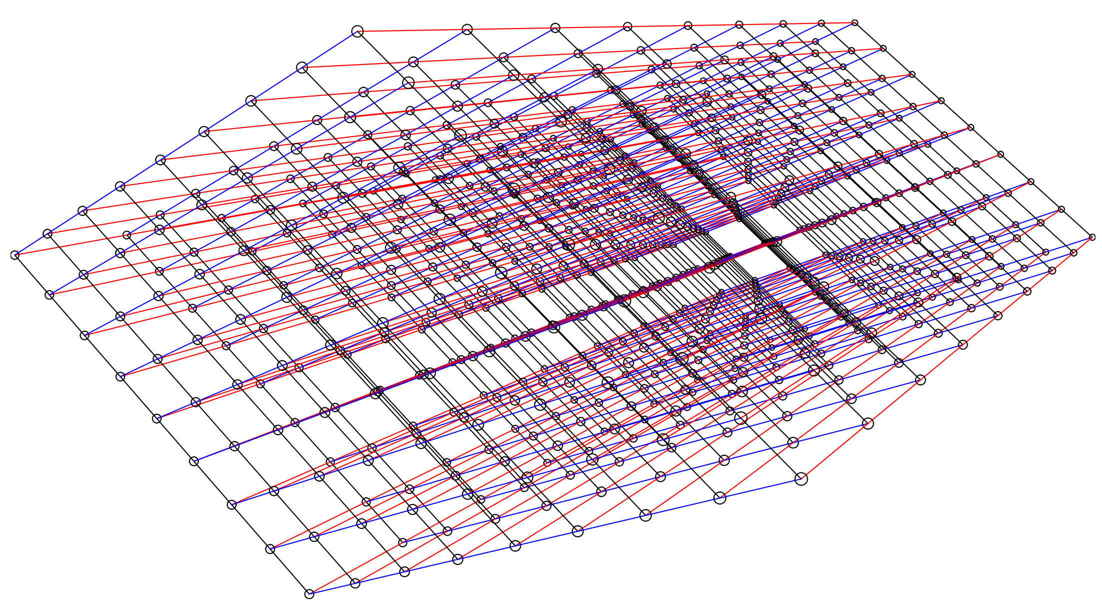

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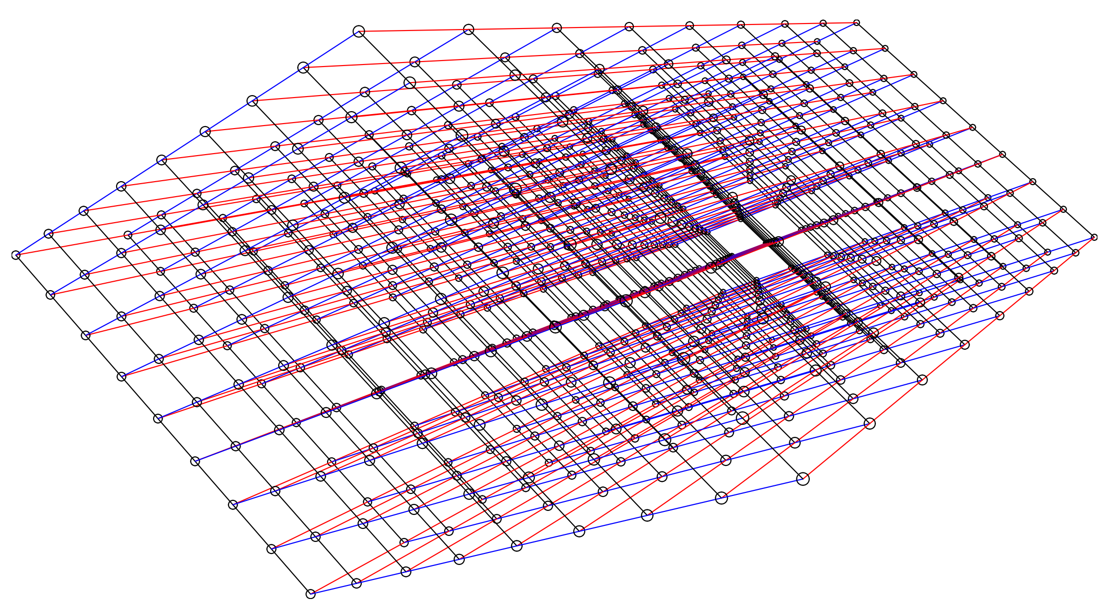

Lattices, mathema

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i.e., $\mathbf{R}b_1 + \ldots + \mathbf{R}$
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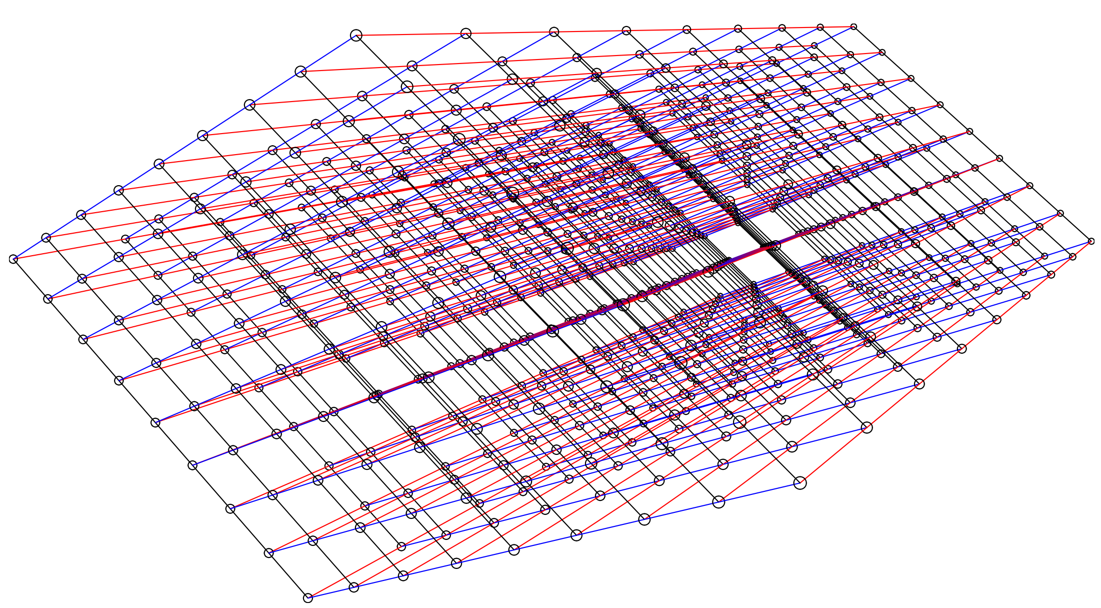

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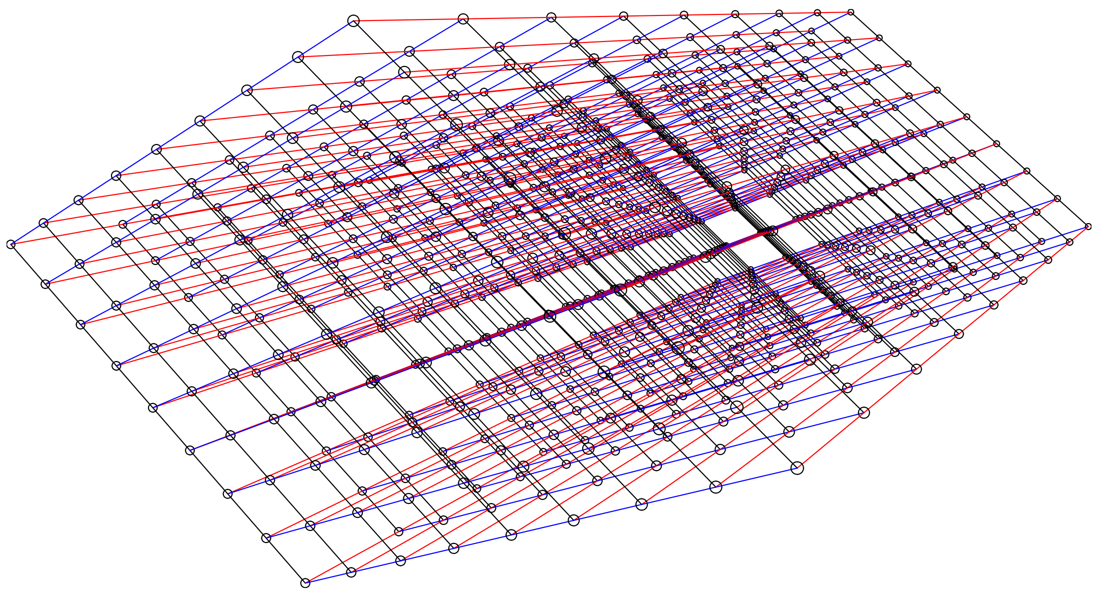

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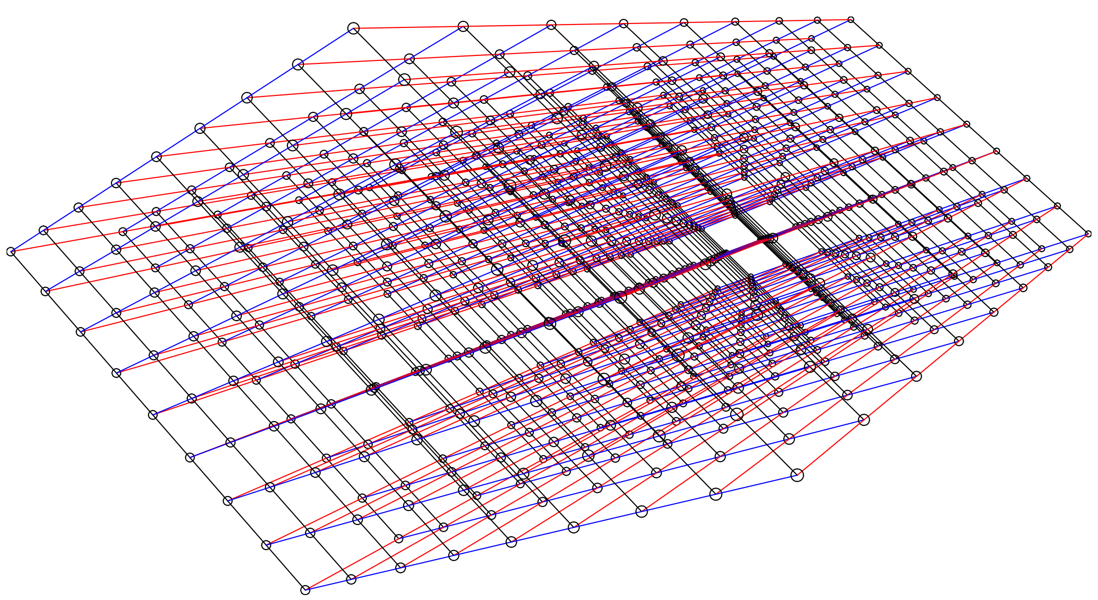

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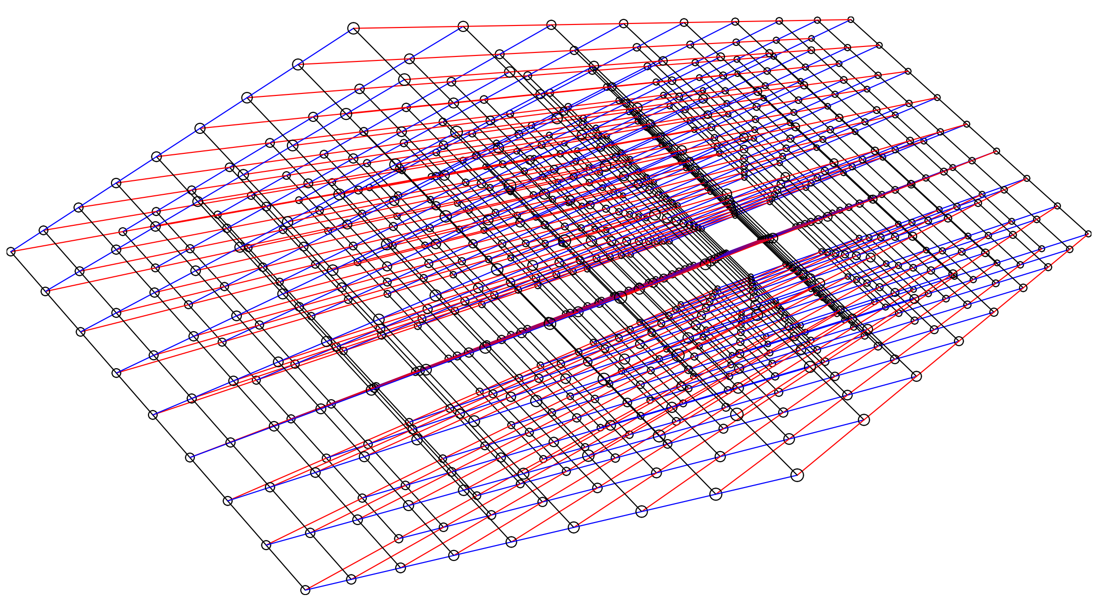

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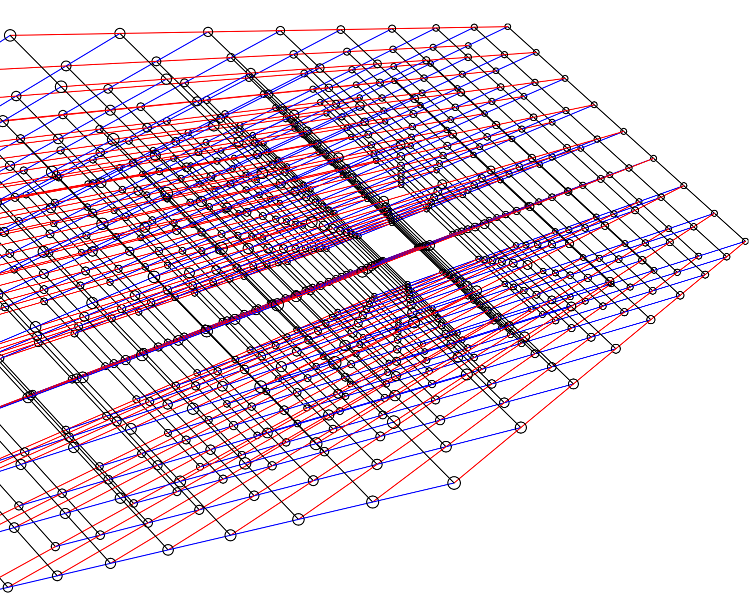

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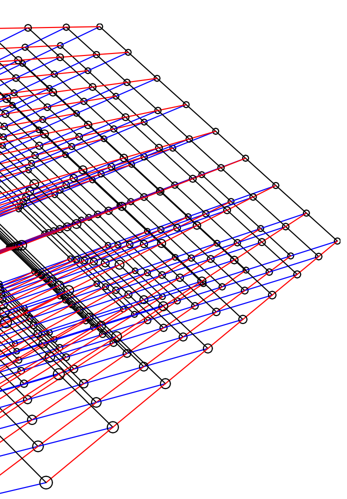

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Compute $A/3 = a$

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Lattice view of NTRU

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Write $A/3$ as
$H_0 + H_1 x + \ldots + H_{n-1} x^{n-1}$.

view of NTRU

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$x^2, \ldots, qx^{n-1}$,

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additions, subtractions.

(a, d) is obtained from

$(q, 0)$,

$(qx, 0)$,

$\vdots$

$(qx^{n-1}, 0)$,

$(A/3, 1)$,

$(xA/3, x)$,

$\vdots$

$(x^{n-1}A/3, x^{n-1})$

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$(a_0, a_1, \ldots$

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$(0, q, \ldots, 0, 0, 0, \ldots$

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$(H_1, H_2, \ldots, H_0, 0$

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is obtained from

$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,

$(0, q, \ldots, 0, 0, 0, \ldots, 0)$,

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$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,

$(H_0, H_1, \ldots, H_{n-1}, 1, 0, \ldots,$

$(H_{n-1}, H_0, \ldots, H_{n-2}, 0, 1, \ldots$

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$(H_1, H_2, \ldots, H_0, 0, 0, \ldots, 1)$

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$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,

$(0, q, \ldots, 0, 0, 0, \ldots, 0)$,

$\vdots$

$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,

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$(H_1, H_2, \ldots, H_0, 0, 0, \ldots, 1)$

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$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots, d_{n-1})$
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$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,
$(0, q, \ldots, 0, 0, 0, \ldots, 0)$,
$\vdots$
$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,
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$(a_0, a_1, \ldots, a_{n-1},$
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$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots, d_{n-1})$
is obtained from
$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,
$(0, q, \ldots, 0, 0, 0, \ldots, 0)$,
$\vdots$
$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,
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$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots,$
is a surprisingly short vector
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$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$ etc.

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balancing: e.g., set up lattice
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Exercise: Describe search for (b, c) as a problem of finding a vector close to a lattice.

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"Product NTRU"
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"Product NTRU" (new nam

2010 Lyubashevsky–Peikert–

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Everyone knows random $G \in R_q$. Alice generates $A = aG + d$ in R_q for small random a, d.

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Alice computes $C - aB$ in R_q, i.e., $m + db + c - ae$ in R_q. Alice reconstructs m, using smallness of d, b, c, a, e.